MVE170 Basic Stochastic Processes

Written exam Thursday 24 March 2011 8.30 am–1.30 pm

Teacher and jour: Patrik Albin, telephone 070 6945709.

Aids: Either two A4-sheets (4 pages) of hand-written notes (xerox-copies and/or computer print-outs are not allowed) or Beta (but not both these aids).

Grades: 12, 18 and 24 points for grades 3, 4 and 5, respectively.

Motivations: All answers/solutions must be motivated.

GOOD LUCK!

Task 1. The integral $\int_0^1 (\sin(1/x))^2\,dx$ can be calculated numerically by means of generating a great number n of independent identically distributed bivariate r.v.'s $\{(X_i,Y_i)\}_{i=1}^n$ that have a common unifom distribution over the unit square with PDF $f_{X_i,Y_i}(x,y)=1$ for $0\le x,y\le 1$ and $f_{X_i,Y_i}(x,y)=0$ elsewhere, and checking how great a fraction of these random numbers that satisfy $(\sin(1/X_i))^2\ge Y_i$. Why does this method give a correct value for the integral as $n\to\infty$? **(5 points)**

Task 2. Consider the random process $X(t)=Y\cos(\omega t+\Theta)$ for $t\in\mathbb{R}$, where Y and Θ are independent r.v.'s that are uniformly distributed over $(-1,1)$ and over $(-\pi,\pi)$, respectively. Find the mean and autocorrelation function of $X(t)$. **(5 points)**

Task 3. A certain product is made by two companies A and B that control the entire market. Currently A and B have 60% and 40%, respectively, of the market. Each year A loses $\frac{2}{3}$ of its market share to B, while B loses $\frac{1}{2}$ of its share to A. Find the percentages of the market that each of A and B hold after 2 years. **(5 points)**

Task 4. Let $Y_n=((1-p)/p)^{S_n}$ for $n\ge 0$, where $S_n=\sum_{i=1}^n X_i$ and $X_1,X_2,\ldots$ are independent identically distributed r.v.'s such that $P(X_i=-1)=1-p$ and $P(X_i=1)=p$ (where $0<p<1$ is a constant). Show that $\{Y_n\}$ is a martingale. **(5 points)**

Task 5. Find the autocorrelation function of a zero-mean contnuous time WSS random process $X(t)$ that has spectral density $S_X(\omega)=N_0/2$ for $|\omega|\le\omega_B$ and $S_X(\omega)=0$ for $|\omega|>\omega_B$ (where $N_0,\omega_B>0$ are constants). **(5 points)**

Task 6. Find the average number of customers in a M/M/1/K queueing system (in equilibrium) when the mean arrival rate λ and the mean service rate μ are equal, $\lambda=\mu>0$. **(5 points)**

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Solutions to written exam Thursday 24 March 2011

Task 1. This is the computational problem for own work of Exercise Session 1.

Task 2. This is a supplementary problem for own work of Exercise Session 2.

Task 3. This is a supplementary problem for own work of Exercise Session 3.

Task 4. This is a supplementary problem for own work of Exercise Session 4.

Task 5. This is a supplementary problem for own work of Exercise Session 5.

Task 6. This is a supplementary problem for own work of Exercise Session 6.