

# MSG800/MVE170 Basic Stochastic Processes

## List of Errata for Hsu's book, Version 13 December 2012

**Equation 2.41.**  $F_X(X)$  should be  $F_X(x)$ .

**Page 60.** On row 6  $|x|$  should be  $\lfloor x \rfloor$ .

**Equation 3.14.** It should be  $\lim_{x \rightarrow \infty}$  instead of  $\lim_{y \rightarrow \infty}$ .

**Figure 3.4.** It should be  $P(Y = 0 | X = 1)$  instead of  $P(Y = 1 | X = 1)$  on the diagonal.

**Problem 4.100.**  $X''$  should be corrected to  $X^n$ .

**Equation 5.28** is wrong as is the proof of (another version of) that formula in Problem 5.25: The second equality in the three line equation on the middle of page 234 is erroneous. To see this we may send  $x_{n-1} \rightarrow \infty$  on both sides of the equation to obtain

$$F_X(x_1, \dots, x_{n-2}, x_n; t_1, \dots, t_{n-2}, t_n) = F_X(x_n; t_n) \times F_X(x_1, \dots, x_{n-2}; t_1, \dots, t_{n-2}),$$

which is to say that  $X(t_n)$  is independent of  $X(t_1), \dots, X(t_{n-2})$ . This is absurd as this need not hold at all for a Markov process (as is e.g., exemplified by a Poisson process).

A correct version of Equation 5.28 is the formula for PMF's given in Problem 5.88.

**Page 210.** The definition of wide-sense stationarity should be the conditions (5.21)-(5.22).

**Equation 5.29.** It should be  $E[X(t_i)]$  instead of just  $E[X(t_i)]$ .

**Page 215.** On row 3 from the bottom (Prob. 5.74) should be (Prob. 5.94).

**Lemma 5.8.1.** It should be  $T_1$  and  $T_2$  instead of  $n_1$  and  $n_2$ , respectively.

**Equation 5.121.** It should be  $\cos \omega\tau$  instead of  $\cos \omega t$ .

**Problem 5.37.** It should be  $\gcd\{2, 4, 6, \dots\}$  instead of  $\gcd\{2, 5, 6, \dots\}$ .

**Problem 5.49.** Four occurrences of  $X(t+\Delta t) - X(0)$  should be  $X(t+\Delta t) - X(t)$ .

**Equation 5.202.** It should be  $\sigma$ 's instead of  $a$ 's in the  $\mathbf{K}_X$ -matrix.

**Problem 5.70.** On row 2 the alternative possible value of  $X_i$  should not be 0 but  $-1$  with probability  $q = 1 - p$ . Such a random variable is not called Bernoulli distributed.

**Problem 5.72.** On row 11  $\frac{n(n-1)}{2}$  should be  $n(n-1)$  and  $\frac{n(n+3)}{2}$  should be  $n(n+1)$ .

**Problem 5.74.** On row 7 it should be  $\frac{n+2-k}{n+2}$  instead of  $\frac{n+2+k}{n+2}$ .

**Problem 5.76.** Change  $E(|g(Xn)|)$  to  $E(|g(X_n)|)$ .

**Problem 5.77.** The calculation on row 4 of the problem should be corrected to

$$\mathbf{E}\{|X_n|\} = \mathbf{E}\{\mathbf{E}\{|X| | F_n|\}\} \leq \mathbf{E}\{\mathbf{E}\{|X| | F_n|\}\} = \mathbf{E}\{|X|\} < \infty.$$

The initial condition should be  $\mathbf{E}\{|X|\} < \infty$  instead of  $\mathbf{E}\{X\} < \infty$  accordingly.

**Problem 5.78.** Theorem 5.82 should be Theorem 5.8.2 at two occurrences.

**Problem 5.79.** On row 8 of page 265 a  $\}$  is missing.

**Page 266.** On row 11  $s_n$  should be  $S_n$  and on row -4 the first  $\frac{a}{a+b}$  should be  $\frac{b}{a+b}$ .

**Problem 5.82.** It should be  $X(t) - \lambda t$  instead of  $x(t) - \lambda t$  at two places.

**Problem 6.8 b.** In the solution it should be  $\partial^2 R_X(s-t)/\partial t \partial s = -d^2 R_X(\tau)/d\tau^2$ .

**Page 288.** On row 8 it should be  $E[X(s)X(\beta)]$  and on row 12 the second  $s^3$  should be  $s^2$ .

**Equation 6.137.** It should be  $R_{X'}(t, s)$  instead of  $R_X(t, s)$ .

**Page 295.** On the last line  $\frac{a}{(a^2+b^2)b}$  should be  $\frac{a}{(a^2-b^2)b}$ .

**Equation 9.20.** In the sum it should be  $(s\rho)^n$  instead of just  $(s\rho)$ .

**Equations 9.22 and 9.36.** Correct  $p$  to  $\rho$  in the former and  $\rho_0$  to  $p_0$  in the latter.

**Equation 9.39.** On the right-hand side  $L_q$  should be divided by  $\lambda_e = \lambda(1-p_K)$ .

**Problem 9.13.** On row 9 (9.16) should be (9.17). In the evaluation of  $W_q$  at the end  $-1/\mu = -3$  has been forgotten in the middle term (but the answer 6.39 min is right).

**Problem 9.16.** The “balance equation” (9.2)  $L = \lambda W$  holds for all steady-state queues provided that we set the total time spent in the system to zero for customers arriving when the system is full for queues with  $K < \infty$ , i.e., if we interpret the fact that customers arriving to a full system bounces away as that they spend zero time in system.

If we instead (as is custom and as is done in the book) define  $W$  as the expected total time spent in the system for customers that really join the system, then we must divide the  $W$  from the previous paragraph by  $1-p_K$  to get this  $W$ . And then we have Equation 9.31  $L = \lambda(1-p_K)W$  for this alternative (and customary) definition of  $W$ .

In Problem 9.16 the sum in the last equation of page 362 should only run to  $K-1$ , giving  $W_q = (L - K p_K)/\mu$ . The sum in first formula on page 363 should also only run to  $K-1$  giving  $W = (L+1 - (K+1)p_K)/\mu$ . Now, here we are dealing with the first definition of  $W$  from above, i.e., we assign zero time in the system for bouncing customers and let that zero influence the mean value  $W$ , so that  $L = \lambda W$  for this  $W$ .

As we want the customary definition of  $W$  to be the expected total time spent in the system for customers that really joins the system we have to divide the  $W$  obtained in the previous paragraph by  $1-p_K$  giving the following correct version of Equation 9.59:  $W = (L+1 - (K+1)p_K)/(\mu(1-p_K))$ . The corresponding correct version of Equation 9.58 is  $W_q = (L+1 - (K+1)p_K)/(\mu(1-p_K)) - 1/\mu = (L - K p_K)/(\mu(1-p_K))$ .

**Problem 9.18.** On row 3 from the bottom  $p^m$  should be  $\rho^m$ .

**Problem 9.29.** Hsu has used the faulty Equation 9.59 (see above) to obtain  $W = (L+1)/\mu = \dots = 0.336$  hours = 20.15 minutes, while a correct answer comes from using the correct version of that formula from above  $W = (L+1 - (K+1)p_K)/(\mu(1-p_K)) = (L - K p_K)/(\mu(1-p_K)) + 1/\mu = 7/57 + 1/6 = 0.289$  hours = 17.4 minutes (or from using Equation 9.31). The answer to task b is  $W_q = W - 1/\mu = W - 10$  minutes = 7.4 minutes.

**Problem 9.30.** The correct answer is 1.

**Table B-2.** In Formula 11 the right hand side should be  $X(\omega) = (\pi/a)e^{-a|\omega|}$ .