MSG800/MVE170 Basic Stochastic Processes

Exercise Session 5

Chapter 9 in Hsu's book

Solved problems. Problems 9.4, 9.13, 9.16, 9.18, 9.19 and 9.20 in Hsu's book.

Problems for own work. Problems 9.21, 9.27, 9.28, 9.29 and 9.30 in Hsu's book.

Computer problem for own work. Find by means of computer simulations an approximation of the probability density function (PDF) of a typical waiting time $W(q)$ (in the queue) to be served for a typical customer arriving to a steady-state M(1)/M(1)/2/4 queue. (In other words, the queue has exp(1)-distributed times between arrivals of new customers as well as exp(1)-distributed service times, and the queue has two servers and two queuing places.)

Note that the sought PDF $f_{W(q)}$ will be of mixed discrete and continuous type $f_{W(q)}(t) = p\,\delta(t) + (1-p)\,f(t)$ for $t \geq 0$, where δ is the Dirac δ-function, p the probability of zero waiting time and $f : [0,\infty) \to [0,\infty)$ is a continuous type PDF (for the waiting time given that it is positive). Answer the task, e.g., by means of supplying a histogram type of approximative plot of f together with the value of p.

Albeit the simulation should in principle be started with the queuing system having a random number of customers according to the steady-state distribution, that is not really important in practice as it is known that the system converges very rapidly to being in steady-state. Hence you can in fact start the simulation with, e.g., 0 or 2 customers in the system. Also note that the expected value $W_q = \mathbf{E}\{W(q)\} = (1-p)\int_0^\infty t\,f(t)\,dt$ of $W(q)$ is given by Equations 9.37 and 9.40 in Hsu's book, so that you can make a basic check of the correctness of your simulation by a simple comparison. In addition, theory gives $p = \mathbf{P}\{W(q) = 0\} = (p_0 + p_1)/(1 - p_4) = 8/11$.