Basic stochastic processes 2007

Take home examination 1

Day assigned: November 13. **Due date:** November 15, 10:00 am

• The take home examination is a strictly individual assignment. Submissions that bear signs of being collective efforts will be disregarded

• Answers without explanations will be disregarded as well.

Problem 1. Consider the Poisson process $\{N(t),\, t \geq 0\}$ of rate λ. Let $S_1,\, S_2,\, \ldots.$ be the arrival times of the process. Fix the time point t and consider the random interval $S_{N(t)+1} - S_{N(t)}$, where we have $S_{N(t)} = 0$, if $N(t) = 0$. What is true for the average length of this interval, $\mu = E[S_{N(t)+1} - S_{N(t)}]$:

(a) $\mu < \frac{1}{\lambda}$,

(b) $\mu = \frac{1}{\lambda}$,

(c) $\mu > \frac{1}{\lambda}$?

Explain your answer. 1p

Problem 2. You need 5 hours to complete a certain routine job. However, your work is interrupted by telephone calls that arrive according to a Poisson process at the rate of 3 calls per hour. The duration of a single telephone call is a random variable which is uniformly distributed between 3 and 7 minutes. How long does it take on the average to complete the job? Explain the model and give a detailed solution. 2p

Problem 3. Customers with items to repair arrive at a repair facility according to a Poisson process with rate λ. The repair time of an item has a uniform distribution on $[a,\, b]$. The exact repair time can be determined upon arrival of the item. If the repair time of an item takes longer than $\frac{a+b}{2}$ time units then the customer gets a loaner for the defective item until the item returns from repair. A sufficiently large supply of loaners is available. In the long run, what is the average number of loaners that are out? Explain the model and give a detailed solution.

Hint. Recognize the $M/G/\infty$ model. 3p

Problem 4. A production process in a factory yields waste that is temporarily stored on the factory site. The amounts of the waste that are produced in successive weeks are independent and identically distributed random variables with finite first two moments μ_1 and μ_2. Opportunities to remove the waste from the factory site occur at the end of each week. The following control rule is used. If at the end of a week the total amount of the waste present is larger than D, then all the waste present is removed; otherwise, nothing is removed. There is a fixed cost of $K > 0$ for removing the waste and a variable cost of $v > 0$ for each unit of excess of the amount D. Describe the regenerative process involved and determine the long-run average cost per time unit. 3p