Take home examination 1 in Basic stochastic processes 2008

Day assigned: November 18, 10:00 am
Due date: November 20, 10:00 am

- The take home examination is a strictly individual assignment. Submissions that bear signs of being collective efforts will be disregarded.
- Students are supposed to give a precise description of the models used to solve the problem and rigorous explanations to the solution.
- Correct answers without proper explanations will be disregarded.
- *Please write the code of the course you are registered in on the upper left corner of the first page of your work.*

Problem 1. Mini group taxis are waiting in a queue for passengers to come. Passengers for those taxis arrive according to a Poisson process with an average of 60 passengers per hour. A taxi departs as soon as two passengers have been collected or 3 minutes have expired since the first passenger has got in the taxi. Suppose you get in the taxi as first passenger. What is your average waiting time for the departure? 3p

Problem 2. Oil tankers with destination Rotterdam leave from harbours in the Middle East according to a Poisson process with an average of two tankers per day. The sailing time to Rotterdam has a Gamma distribution with an expected value of 10 days and a standard deviation of 4 days. Estimate the probability that the number of oil tankers that are under way from the Middle East to Rotterdam at an arbitrary point of time exceeds 30. 3p

Problem 3. The bus that takes you home from Chalmers arrives at the nearest bus stop from early morning till late in the evening according to a renewal process with interarrival times that are uniformly distributed between 5 and 10 minutes. You come to the bus station at 5 pm. Estimate your average waiting time for the bus to arrive. 3p

Problem 4. Messages arrive at a communication channel according to a renewal process with a mean interarrival time $1/\lambda$. The messages are stored in a buffer. The buffer is emptied immediately after M messages have accumulated.

The time needed to empty the buffer is negligible. A holding cost of $h > 0$ is incurred for each time unit the message has to wait in the buffer and a fixed cost of $K > 0$ is incurred for each time the buffer is emptied. Determine the long-run average cost per time unit and find a value of M that minimizes this cost. 3p

Problem 5. Consider a Poisson arrival process with rate λ. For each fixed $t > 0$ define the random variable δ_t as the time elapsed since the last arrival before t and set $\delta_t = t$ if there are no arrivals before t. Compute the distribution function of δ_t and sketch its graph. 3p