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Applied Mathematics and Statistics
Chalmers and GU

MVE055 / MVE051 / MSG810 Matematisk statistik och diskret matematik

Exam 29 August 2018, 14.00 - 18:00

Allowed aids: Chalmers-approved calculator
and one (two-sided) A4 sheet of paper with your own notes.
Total number of points: 30. To pass, at least 12 points are needed.
Note: All answers should be motivated.

1. **(5 points)** Consider a discrete random variable X such that $P(X = k) = \frac{1}{4}$, for $k \in \{-3, -1, 1, 3\}$.
   (a) Compute the covariance between the two random variable X and $Y = X^2$.
   (b) Based on your previous findings, are X and Y independent?
2. **(5 points)** State and prove Chebychev's inequality.
3. **(5 points)** Let X be a random variable taking values in the natural numbers $\{1, 2, 3, ...\}$. Suppose that X satisfies the following property
$$P(X \geq x + 1 | X \geq x) = 1 - p, \text{for all } x = 1, 2, 3, ...$$
   for some $p \in [0, 1]$.
   (a) Show that $\frac{P(X \geq x+1)}{P(X \geq x)} = 1 - p$ for all $x = 1, 2, 3, ...$
   (b) Use the previous point and recursion to show that, for all $x = 1, 2, 3, ..., P(X \geq x+1) = (1 - p)^x$.
   (c) Find $P(X = x)$ for all $x = 1, 2, 3, ...$ What is the distribution of X?
4. **(5 points)** Let U be a uniformly distributed random variable on the interval $[a, b]$. Assume $t > 0$ and s are two constants. Which of the following statements are true? (Motivate your answer for each statement)
   (a) The random variable $V = s + tU$ is uniformly distributed in $[s + at, s + tb]$.
   (b) The random variable $V = s + tU$ is uniformly distributed in $[(s + a)t, (s + b)t]$.
   (c) The expected value of $V = s + tU$ is $s + t\left(\frac{b-a}{2}\right)$.
5. **(5 points)** Let $X_1, ..., X_n$ be independent and identically distributed random variables with normal distribution $N(\mu, \sigma^2)$. Assume that the variance σ^2 is known. Denote by I_α the $100(1 - \alpha)\%$ standard confidence interval for the mean μ.

(a) What happens to the width of the confidence interval when α increases?

(b) What happens to the width of the confidence interval when σ^2 decreases?

(c) What happens to the width of the confidence interval when the sample size n doubles?

6. **(5 points)** A gambler can play a game of chance in which at every play he can win 1 kr with probability 0.25 or lose 1 kr with probability 0.75. The player starts with a capital of $k = 2$ kr and he stops playing whenever he has a total of 3kr. Let X_n denote the capital of the gambler after n games, and $X_0 = 2$.

   (a) Write down the transition matrix P of the Markov chain $\{X_n\}_{n\in\mathbb{N}}$.

   (b) Find the probability that the gambler will go bankrupt.