Assignment 1

Differential Equations and Scientific Computing Kb2, TMA 205, part A

General

This is the assignment for the first part (Week 1–4) of the course which covers the solution of one-dimensional boundary value problems using the finite element method. In the second part (Week 4–8) we basically repeat the same procedure for two-dimensional problems. Thus a good understanding of the first part of the course will lead to a pleasant second part!

You are asked to hand in a written documentation of your work. Remember that it is always important that your *written work is neat and easily accessible.*

Your work is individual and each student hands in their own documentation. However, it is natural that you work together in pairs of two students.

Problems marked by * are primarily intended for students aiming at higher grades (4 or 5). It is not compulsory to hand in any of the * problems, but you may of course do so if you like. Your grade will be based on your result on the written exam, and will not be affected by whether or not you hand in any * problems. Note, however, that some of the questions on the written exam will be based on * problems, so in order to achieve higher grades you need to work also with these problems.

Finally, please use the cover sheet included in the end of the assignment.

Deadline: Wednesday, September 25 (Part A – C)[1]
Monday, September 30 (Part D)

A. My solver for two-point boundary value problems

1. Choose a function f defined on $(0,1)$ and a partition of $(0,1)$ with mesh function $h(x)$; calculate the L_2-projection[2] $P_h f$ of f. Plot f, the (nodal) interpolant $\pi_h f$, and the L_2-projection $P_h f$ in the same diagram. Refine the mesh a few times and examine what happens with the accuracy of your approximations.

2. Write a *finite element solver* for two-point boundary value problems of the form

$$-(au')' + cu = f, \quad \text{in } (0,1), \tag{1}$$

(where $a(x) > 0$ and $c(x) \geq 0$) with Robin boundary conditions. Start from the template program `MyFirstPoissonSolver`[3]. Make sure you understand the ba-

[1] Hand in these parts in connection with the midterm exam ("deltentamen").

[2] We will consider this problem in studio session 2: *Kvadratur. L2-projektion.*

[3] Information about how to obtain this program will be given in the instructions for studio session 3: *Styvhetsmatris. Robinrandvillkor.*

sic steps in the finite element main program as well as the details of the function `MyFirstPoissonAssembler`.

3. (a) Solve a problem of type (1) analytically and verify that your solver approximates the analytical solution when the mesh size decreases.
   (b*) Write a program which calculates the energy norm of the error and verify the *a priori* error estimate using a log-log diagram.

4. Extend your solver to the *time dependent* problem[4]

$$u_t - (au')' + cu = f, \quad \text{in } (0, 1),$$

   where u_t denotes the time derivative.

5*. Extend to $-(au')' + bu' + cu = f$, with the *convection term* bu'.

6*. Include *adaptivity* based on an a posteriori error estimate for the stationary problem.

Hand in: a short documentation of your work.

B. Problems

1. Solve the problems ("*Räkneuppgifter*") handed out each week. Note that some of these problems also may be marked by *: cf. the remark in section *General*.

Hand in: – (do not hand in solutions to the problems)

C. Topics presentations

For each topic below, write a short summary, including basic ideas and theoretical results. Make sure you understand all basic notations and steps in proofs and derivations. Working on these presentations gives you an overview of the theory, and is a good way to prepare yourself for the written exam.

1. The vector space of continuous piecewise linear polynomials V_h on a partition of an interval. Representation formula of functions $v \in V_h$ as a linear combination of hat basis functions.

2. Approximation of a given function by interpolation. Definition and error estimates including proofs.

3. Approximation of given function by L_2-projection: definition, derivation of linear system of equations, algorithm, existence, uniqueness, and error estimate.

[4] We will do this in studio session 4: *Tidsberoende problem.*

4. Numerical quadrature: purpose, basic examples, and accuracy.

5. Approximation of solutions to differential equations by the finite element method: formulation, derivation of discrete system of equations, algorithm, existence and uniqueness.

6. Boundary conditions: Neumann, Robin, and non-homogeneous Dirichlet conditions.

7. A priori error estimate in the energy norm (with homogeneous Dirichlet boundary conditions).

8. (a) *Formulate* an a posteriori error estimate in the energy norm (with homogeneous Dirichlet boundary conditions). (b*) *Prove* the a posteriori error estimate.

9. Formulate a finite element method for the time dependent problem based on finite elements in space and backward Euler timestepping.

10*. A priori and a posteriori error estimates in the energy norm with Robin boundary conditions.

11*. Explain what an adaptive algorithm is and how a posteriori error estimates are used.

Hand in: a short readable summary of each topic (without too much detail).

D. Application

1. Use your finite element solver to solve the example application (handed out during LV3) or solve a problem of your own interest.

2*. Present a second application to a problem of interest.

Remember that you can do stationary, time dependent, and also eigenvalue problems.

Hand in: a presentation of your work including the statement and motivation for the problem, mathematical modeling, and computational results.

Assignment 1

Name:

Date:

Coworker:

Problems solved

A1	A2	A3.a	A3.b*	A4	A5*	A6*

C1	C2	C3	C4	C5	C6	C7.a	C7.b*	C8	C9	C10*	C11*

D1	D2*