

COMPLEMENTARY EXERCISES ON ORDINARY DIFFERENTIAL EQUATIONS

1. Solve the following initial value problems (hint: integrating factor)

$$(a) \quad u'(x) - 4u(x) = 0, \quad u(0) = 1$$

$$(b) \quad y'(x) + 2y(x) = 1, \quad y(0) = 2$$

$$(c) \quad \dot{u}(t) = -3u(t) + 5, \quad u(1) = 1$$

2. Solve the following initial value problems (hint: integrating factor)

$$(a) \quad xy'(x) + 3y(x) = x^2, \quad y(2) = 1$$

$$(b) \quad y'(x) = y(x) - xe^{-x}, \quad y(0) = 2$$

3. Solve the following initial value problems

$$(a) \quad (x^2 + 4)u'(x) - 4xu(x) = x, \quad u(1) = 1$$

$$(b) \quad \cos^2 t \dot{y}(t) + y(t) = 0, \quad y(0) = 1$$

$$(c) \quad (x + 1)y'(x) + 2y(x) = (1 + x)^2, \quad y(0) = 0$$

4. Solve the following initial value problems

$$(a) \quad u'(x) = \frac{1}{2}(x + u(x)), \quad u(1) = 1$$

$$(b) \quad xu'(x) - u(x) = x \sin x, \quad u(\pi) = 0$$

$$(c) \quad \frac{dy}{dx} + 2xy = xe^{-x^2}, \quad y(0) = 0$$

Answers:

$$1. \quad (a) \quad u(x) = e^{4x} \quad (b) \quad \frac{1}{2}(3e^{-2x} + 1) \quad (c) \quad \frac{1}{3}(5 - 2e^{-3t+3})$$

$$2. \quad (a) \quad y(x) = \frac{x^5 + 8}{5x^3} \quad (b) \quad \frac{1}{4}(1 + 7e^{2x} + 2x)e^{-x}$$

$$3. \quad (a) \quad u(x) = \frac{1}{20}(11 + 8x^2 + x^4) \quad (b) \quad y(t) = e^{-\tan x}$$

$$(c) \quad y(x) = \frac{4x + 6x^2 + 4x^3 + x^4}{4(x + 1)^2}$$

$$4. \quad (a) \quad 4e^{-1/2+x/2} - x - 2 \quad (b) \quad x \int_{\pi}^x \frac{\sin s}{s} ds \quad (c) \quad \frac{x^2}{2}e^{-x^2}$$