Random tessellations for modelling of sintered structures

Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

Department Models and Algorithms in Image Processing

Claudia Lautensack

The sinter process

Initial state

powdered or granular material is compressed in a die

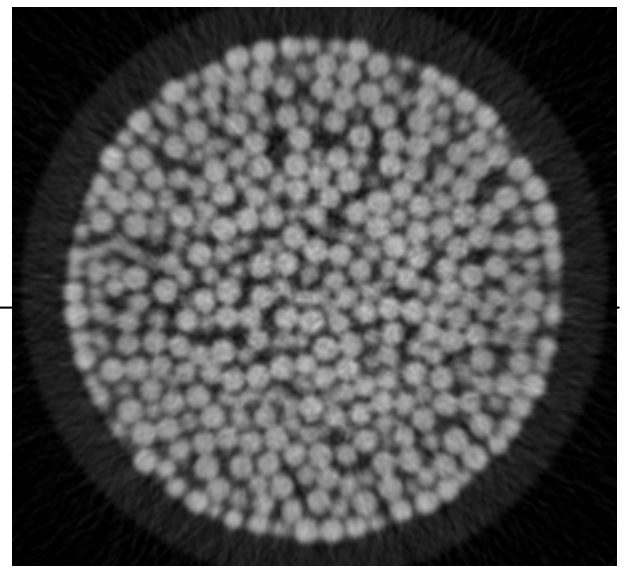
Intermediate states

material is heated to a temperature close to but below the melting point

=> particles stick together resulting in a porous material

Final state

space completely filled by the material



Modelling of sintered structures

Project

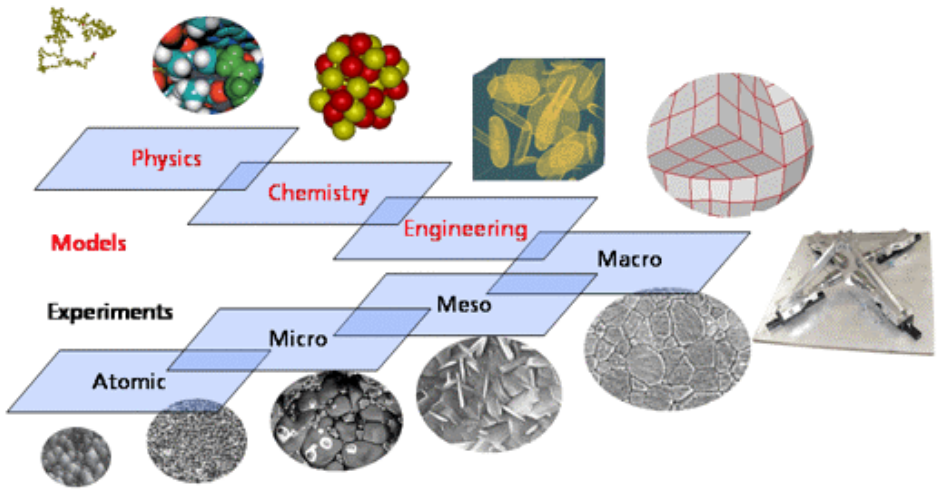
MAVO MMM-Tools (multiscale materials modelling)

AVO MMM-Tools

Fraunhofer internal project
(9 institutes)

Focus

development of a method for the characterization of materials on all scales of temporal and spatial resolution



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Modelling of sintered structures

Our task

geometric modelling of sintered materials in all stages of the sintering process

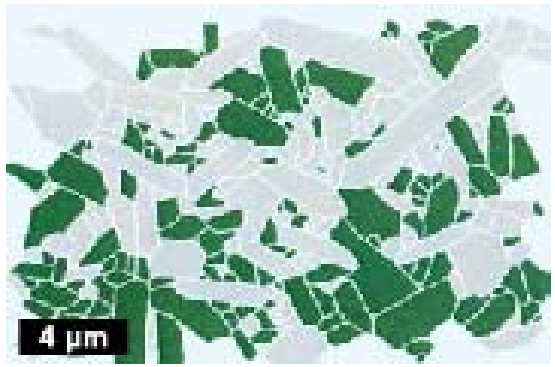
=> development of a structure generator for sintered materials

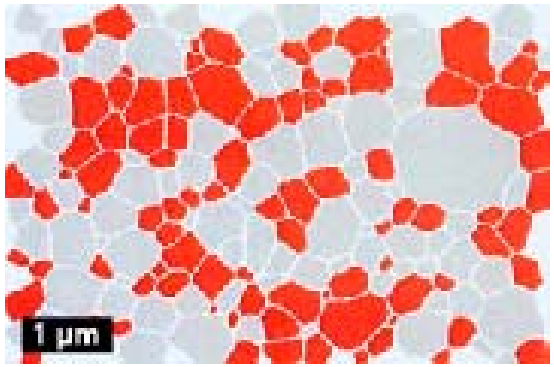
Problems

- samples from the same material may look quite different depending on conditions during sintering
- find a geometric model for arbitrary states without simulation of the physical process (too complicated)
- modelling of rare events e.g. defects

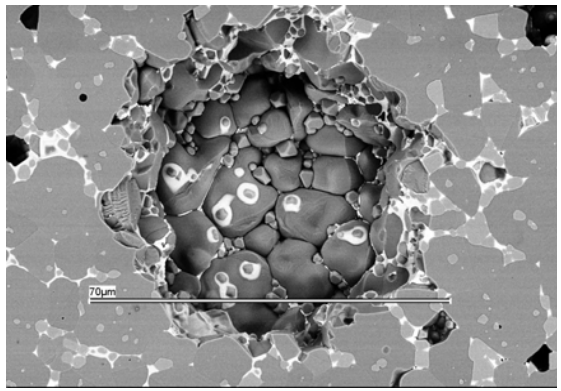
Examples

Different structures of Alumina crystals





Defect



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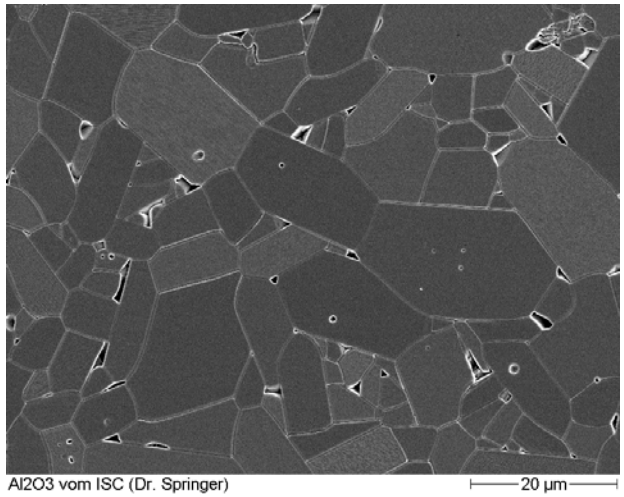
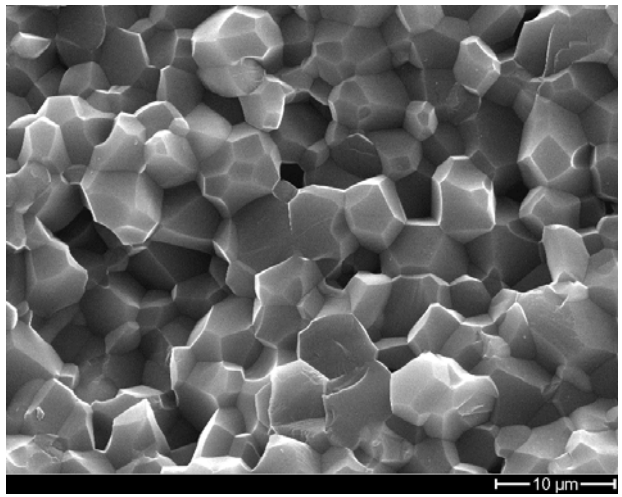
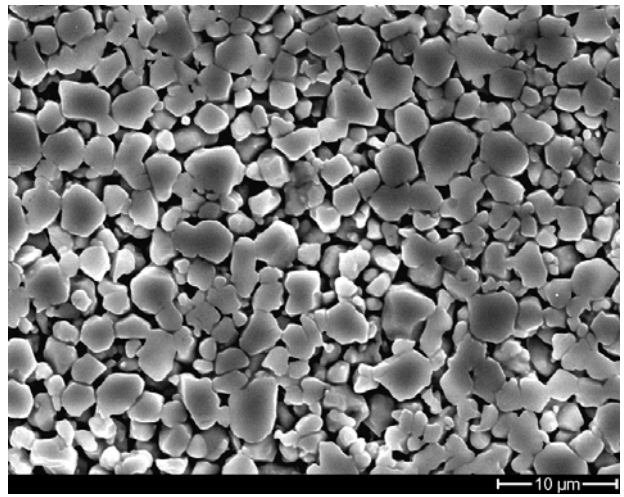
Examples

Fracture surfaces and cuts of sinter ceramics







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Models for the sinter process

Initial state	=> (dense) packing of particles
Intermediate states	=> incomplete tessellation
Final state	=> (random) tessellation

The initial state – Packing of particles

Desired

(dense) packing of particles

Possible solutions

Force biased algorithm (used at the moment)

- Volume density of approximately 64%
- No analytic description

Marked hard core point field

- Not yet clear which model to use

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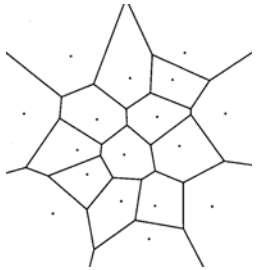
The final state - Tessellations

Voronoi tessellation

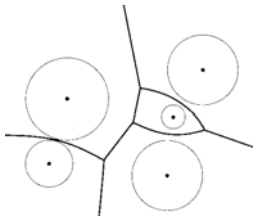


Given: $\mathcal{P} = \{p_i \,|\, i \in I\} \subset \mathbb{R}^d$ set of points

Cells: $C(p_i) = \left\{x \in \mathbb{R}^d \,|\, d(x, p_i) \leq d(x, p_j)\, \forall p_j \in \mathcal{P}\right\}$

Problem: size of cells does not correspond to size of particles

Johnson-Mehl tessellation



Given: $\mathcal{P} = \{(p_i, r_i) \,|\, i \in I\} \subset \mathbb{R}^d \times \mathbb{R}$ set of weighted points

Cells: $C(p_i, r_i) = \left\{x \in \mathbb{R}^d \,|\, d(x, p_i) - r_i \leq d(x, p_j) - r_j\, \forall (p_j, r_j) \in \mathcal{P}\right\}$

Problem: complicated shape of cells

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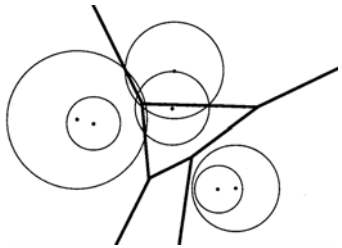
The final state - Tessellations

Laguerre tessellation



Given: $\mathcal{P} = \{(p_i, r_i) \mid i \in I\} \subset \mathbb{R}^d \times \mathbb{R}^+$ set of weighted points

Cells: $C(p_i, r_i) = \left\{x \in \mathbb{R}^d \mid d(x, p_i)^2 - r_i^2 \leq d(x, p_j)^2 - r_j^2 \, \forall (p_j, r_j) \in \mathcal{P}\right\}$

- d=3: Every simple convex cellular structure is a Laguerre tessellation. (Davis 1959)
- realistic in shape and size of cells
- used in many applications, but not yet treated analytically

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Applications of Laguerre tessellations

Modelling of

- polycrystalline materials
- foams
- biological tissues
- molecular surfaces (proteins)

Algorithms for

- construction
- motion of points
- elementary topological transformations (disappearance of cells, change of neighbours)

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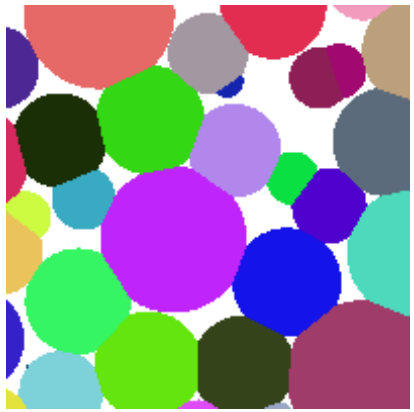
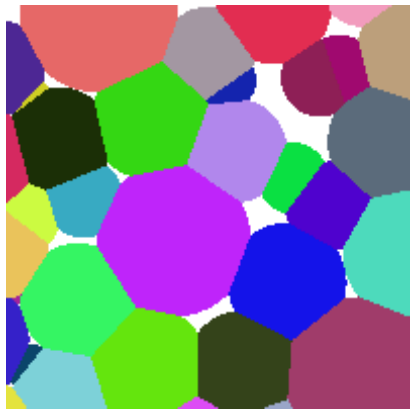
Model for different states

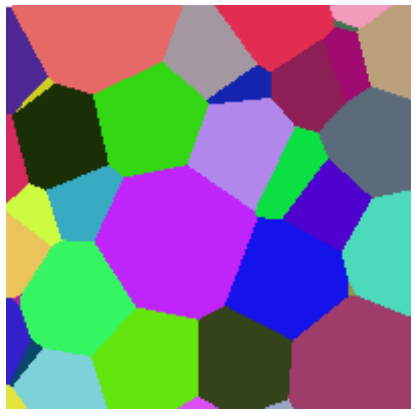
Initial state: 200 balls in a cube with 200 pixels side length created by force biased algorithm

initial state





intermediate states



final state

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Model for different states

Initial state: 200 balls in a cube with 200 pixels side length created by force biased algorithm

Interesting questions

adequate model for initial state

replace force biased algorithm with marked (Gibbs) point process

properties of random Laguerre tessellations

cell characteristics

probability for empty cells

=> easiest case: marked Poisson point process,
interesting for applications: Hard core point process

limits of the Laguerre model in applications

more general particles (platelets, polyhedra, ...)

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