Space-time growth-interaction models in modelling tree growth

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Outline

- Immigration-growth spatial interaction process
- Growth models and interaction models
- Estimation of the parameters of the full space-time model
- Modelling growth of pine trees
- Conclusions/Future work

Immigration-growth-spatial interaction process

New immigrants (trees) arrive according to a Poisson process, with rate α, have uniformly distributed locations, and are assigned marks from $U(0, \epsilon)$, $\epsilon > 0$ (small).

In small time intervals $(t, t + dt)$, each individual either dies 'naturally' with probability μdt, or undergoes the deterministic size change

$$
\begin{aligned}
m_i(t + dt) &= m_i(t) + f(m_i(t))dt \\
&\quad + \sum_{j \neq i} h(m_i(t), m_j(t), \|x_i - x_j\|)dt,
\end{aligned}
$$

where

$f(\cdot)$ individual growth function

$h(\cdot)$ spatial interaction function

$\|x_i - x_j\|$ distance between trees i and j

Growth functions

- Logistic growth function

$$f(m_i(t)) = \lambda m_i(t)(1 - m_i(t)/K),$$

where λ is the intrinsic growth rate and K the carrying capacity

- Linear growth function

$$f(m_i(t)) = \lambda(1 - m_i(t)/K)$$

- The linear growth function preferable if an established upper and lower canopy structure desired

Interaction functions

- Symmetric interaction function

$$h(m_i(t), m_j(t), \|x_i - x_j\|) \\ = -bI(\|x_i - x_j\| < r(m_i(t) + m_j(t)),$$

where $b > 0$ and $I(x)$ denotes the indicator function (see e.g. Renshaw and Särkkä, 2001).

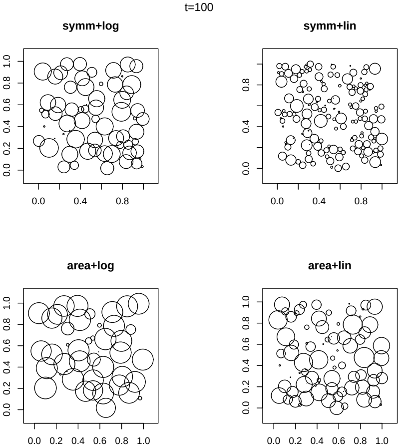
- Non-symmetric area-interaction function

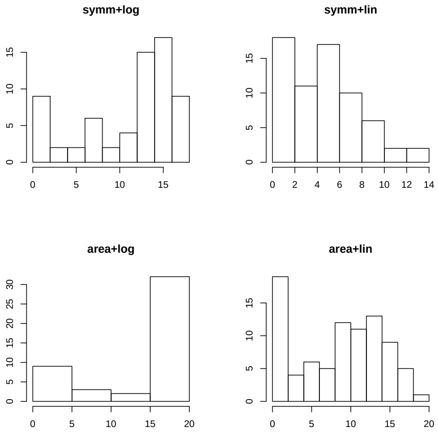
$$h(m_i(t), m_j(t), \|x_i - x_j\|) \\ = -b \frac{|D(x_i, rm_i(t)) \cap D(x_j, rm_j(t))|}{\pi r^2 m_i^2(t)},$$

where $|\cdot|$ denotes area and $D(x, m)$ a disk with radius m centered at x (see Gerrard, 1969; Särkkä and Renshaw, 2006).

- Both models are useful in forestry

Comparison of the models




symm+log
symm+lin
area+log
area+lin
0
5
10
15
0
2
4
6
8
10
12
14
20
25
30

Full space-time estimation

- α is estimated by the total number of immigrants divided by the length of the time interval
- For μ we use the ML estimator

$$\hat{\mu} = n_T//[\sum_{i=1}^{n_T} t_i + \sum_{j=1}^{m_T} s_j],$$

where $t_1, \ldots, t_{n_T}$ denote the lifetimes of the n_T individuals who have died from natural causes by time T and $s_1, \ldots, s_{m_T}$ the lifetimes of the m_T individuals who are still alive at time T.

- Least squares estimates for λ, K, r and b by minimizing (w.r.t. the parameters)

$$S = \sum_{t=1}^{T-1} \sum_{i \in \Omega_t} (\tilde{m}_i(t+1) - m_i(t+1))^2$$

Parameter estimates

The symmetric model with logistic growth

	α	μ	λ	K	r	b
true	5.0	0.02	1.0	20	0.005	1.5
mean	1.9	0.012	0.93	18	0.0048	0.96
e.s.e.	0.03	0.0002	0.08	0.5	0.0002	0.10

The area-interaction model with linear growth

	α	μ	λ	K	r	b
true	5.0	0.02	1.0	20	0.005	1.5
mean	2.5	0.014	0.99	21	0.0056	0.64
e.s.e.	0.02	0.0002	0.03	0.2	0.0001	0.08

In terms of precision the LS procedure works well but most of the estimates are biased downwords

Sensitivity study

Now, the generating process is the symmetric interaction model with logistic growth and we fit the area-interaction process with linear growth to the generated data.

	α	μ	λ	K	r	b
true	5.0	0.02	1.0	20	0.005	1.5
mean	1.9	0.013	3.5	19	0.0064	0.80
e.s.e.	0.03	0.0004	0.3	0.6	0.0006	0.1

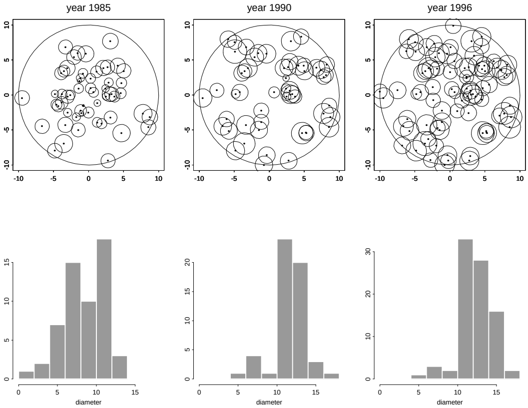
Conclusions

- Estimates are biased downwords (large sampling intervals)
- Standard errors of the estimates are small
- The procedure works well in determining the presence of a growth-interaction process even if an "incorrect" model is fitted

Swedish pine data

- The sample area is located in a developing woodland near Stockholm
- Data: tree location and d.b.h. (diameter at breast height) recorded in a circular plot of radius 10m on three different occasions in 1985, 1990 and 1996. All trees greater than 1.3m in height or 10cm in diameter are included
- Exact arrival times of the trees are unknown. They are taken uniformly distributed on the time interval they arrived.
- We fitted two models to the data: symmetric interaction with logistic growth and area-interaction with linear growth

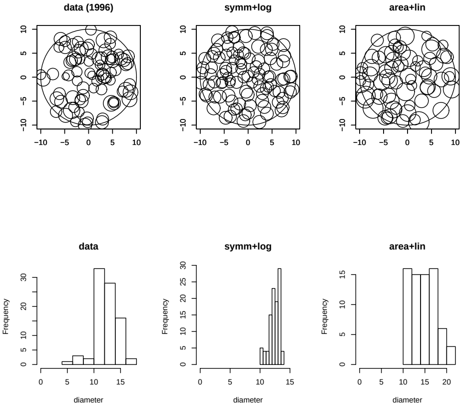
Data



Parameter estimates

Model	$\hat{\alpha}$	$\hat{\mu}$	$\hat{\lambda}$	$\hat{K}$	$\hat{r}$	$\hat{b}$
symm+log	5.0	0.0119	3.08	14.2	0.097	1.12
area+lin	5.0	0.0119	0.478	96.4	0.178	0.044

Data and simulated tree patterns (1996)



Final conclusions

- Given the results of the full space-time estimation procedure, we think we have a promising technique for space-time estimation which seems to be of interest for foresters
- Even though the full space-time estimation approach seems to work ok even if an "incorrect" model is fitted, one should choose the model by using all prior ecological, physical, etc. information on the process that is available.
- (Downwords) biasedness of the estimates is of concern.
- Computational aspects: maximum likelihood estimation, edge effects
- Find more reasonable models for trees
  - randomness into the growth process
  - mixed (multi-species) forests