Recursive Estimation of Mixture Models with Applications in Video Analysis

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Smögen August 22, 2006

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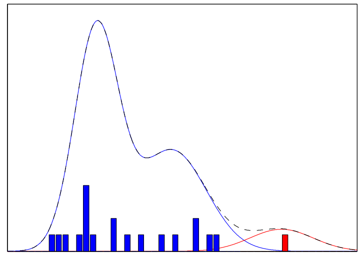
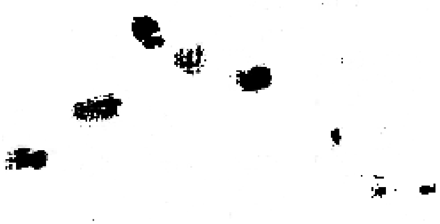
- Standard solution is to model each pixel using a Gaussian mixture.
- Update the parameters of each Gaussian mixture using K-means approximation.

(Stauffer & Grimson, 1999; Power & Schoones, 2002; et al.)



Background



- Part of a project to assess the safety of intersections by monitoring driving patterns.
- The project uses vehicle tracking based on a foreground/background segmentation.
- The tracking has experienced problems due to bad segmentation. Thus we want to improve the segmentation.

Gaussian Mixture Model

Introduce a model with local, per pixel, background and a global foreground.
Using GMMs the distributions of the pixel values, x_{ti}, becomes,

$$p(x_{ti}) = \pi^F \sum_k \pi_k^F p(x_{ti} \mid \mu_k^F, \Sigma_k^F) + (1-\pi^F) \sum_l \pi_{il}^B p(x_{ti} \mid \mu_{il}^B, \Sigma_{il}^B) \, .$$

Our goal is to estimate the parameters and classify each x_{ti}.
But parameters might not be constant over time and the data arrives sequentialy, i.e. we need to recursivly update the parameters.

Gaussian Mixture Model – Simplified

$$p(x_{ti}) = \pi^F \sum_k \pi_k^F p(x_{ti} \mid \mu_k^F, \Sigma_k^F) + (1-\pi^F) \sum_l \pi_{il}^B p(x_{ti} \mid \mu_{il}^B, \Sigma_{il}^B)$$

To illustrate a simpler case is used,

$$p(x_{ti}) = \sum_k \pi_k p(x_{ti} \mid \mu_k, \Sigma_k) \ .$$

Probability that pixel x_{ti} belongs to the k^{th}–mixture,

$$P_{tik} = \frac{\pi_k p(x_{ti} \mid \mu_k, \Sigma_k)}{\sum_k \pi_k p(x_{ti} \mid \mu_k, \Sigma_k)} \ .$$

Bayesian Formulation

Let $\theta_T = \{\pi_T, \mu_T, \Sigma_T\}$. Ideally we want a recursive Bayesian formulation,

$$p(x_{Ti}) = p(x_{Ti} \mid \theta_T) p(\theta_T \mid x_{<T}) \ .$$

An approximation is,

$$p(x_{Ti}) = p(x_{Ti} \mid \theta_T) p(\theta_T \mid \Psi_T) \ ,$$

where

$$\Psi_T = g(\theta_{T-1}, \cdots, \theta_1) = \tilde{g}(\theta_{T-1}) \ .$$

But how do we select $\Psi_T = \tilde{g}(\theta_{T-1})$?

Bayesian Formulation – Updating Equations

$$\ln L = \sum_{i=1}^{N} \ln p(x_{Ti} \mid \theta_T) + \ln p(\theta_T \mid \Psi_T)$$

Suitable priors (Gelman et.al., 2004; Ormoneit & Tresp, 1998):

$$\begin{aligned} \pi_{1\cdots K} &\sim \mathrm{D}(\beta_1, \cdots, \beta_K) \ , \\ \mu_k \mid \Sigma_k &\sim \mathrm{N}(m_k, {\eta_k}^{-1}\Sigma_k) \ , \\ \Sigma_k &\sim \mathrm{IW}(\xi_k, V_k) \ . \end{aligned}$$

Using these priors the updating equation for μ becomes,

$$\mu_{Tk} = \frac{m_k \eta_k + \sum_i x_{Ti} P_{Tik}}{\eta_k + \sum_i P_{Tik}} \ .$$

But how do we select $\Psi_T = \tilde{g}(\theta_{T-1})$?

Offline Estimates

Introduce a forgetting factor in the log-likelihood to handle varying parameters:

$$\ln L = \sum_{t=1}^{T} \sum_{i=1}^{N} \alpha^{T-t} \ln p(x_{ti} \mid \theta) \ .$$

The EM-algorithm gives offline parameter estimates,

$$\mu_k = \frac{\sum_{t,i} x_{ti} P_{tik} \alpha^{T-t}}{\sum_{t,i} P_{tik} \alpha^{T-t}} \ .$$

Offline Estimates cont.

Offline parameter estimate,

$$\mu_k = \frac{\sum_{t,i} x_{ti} P_{tik} \alpha^{T-t}}{\sum_{t,i} P_{tik} \alpha^{T-t}} .$$

Introduce the cumulative sums,

$$S_{Tk} = \sum_{i=1}^{N} \sum_{t=1}^{T} P_{tik} \alpha^{T-t} = \alpha S_{T-1,k} + \sum_{i=1}^{N} P_{Tik} ,$$

the offline estimates can be rewritten as online updating equations,

$$\mu_{Tk} = \frac{\mu_{T-1,k} \alpha S_{T-1,k} + \sum_i x_{Ti} P_{Tik}}{\alpha S_{T-1,k} + \sum_i P_{Tik}} .$$

Algorithm

1. Calculate posterior probabilities (P_{Tik}).
2. Recursively update the parameters.
3. Introduce new foreground components.
4. Transfer components representing stationary pixels from foreground to background.
5. Remove old, seldomly observed components.

Selecting the Priors

Comparing the updating equations,

$$\mu_{Tk} = \frac{m_k \eta_k + \sum_i x_{Ti} P_{Tik}}{\eta_k + \sum_i P_{Tik}} ,$$

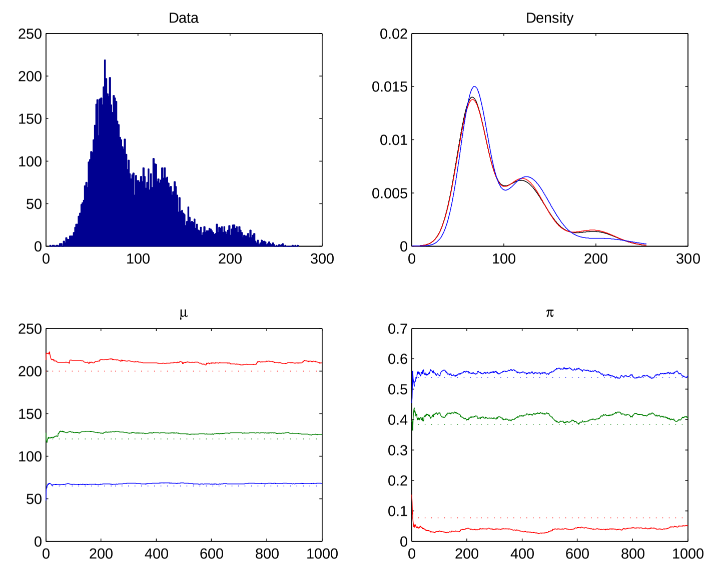
and

$$\mu_{Tk} = \frac{\mu_{T-1,k} \alpha S_{T-1,k} + \sum_i x_{Ti} P_{Tik}}{\alpha S_{T-1,k} + \sum_i P_{Tik}} .$$

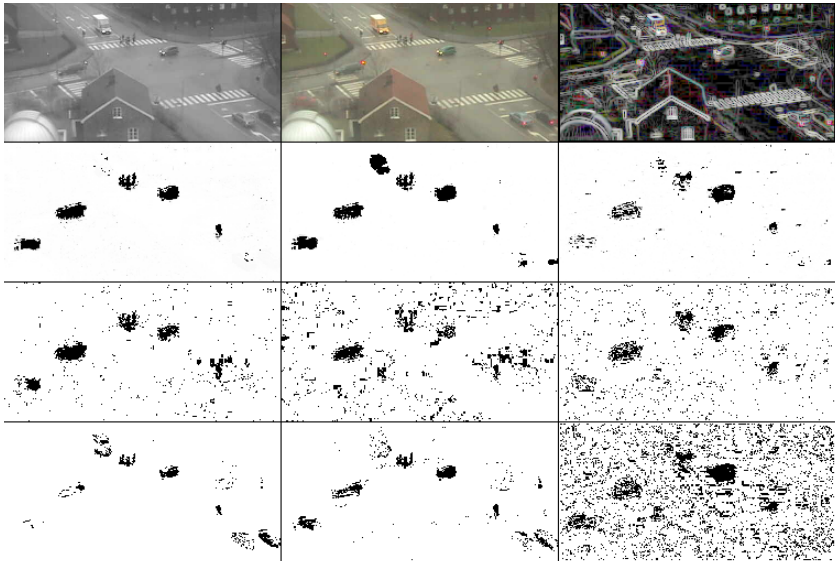
Gives a way of selecting the priors, e.g.

$$\pi_{T,1\cdots K} \sim \mathrm{D}(\alpha S_{T-1,1} + 1, \cdots, \alpha S_{T-1,K} + 1) ,$$
$$\mu_{Tk} \mid \Sigma_{Tk} \sim \mathrm{N}(\mu_{T-1,k}, (\alpha S_{T-1,k})^{-1} \Sigma_{Tk}) .$$
$$\Sigma_{Tk} \sim \mathrm{IW}(\alpha S_{T-1,k} - d - 2, \alpha S_{T-1,k} \Sigma_{T-1,k}) ,$$

Simulated Data



Segmentation



Conclusions and Future Work

- Recursive parameter estimates in a Gaussian Mixture model.
- Bayesian interpretation of the recursive estimates.

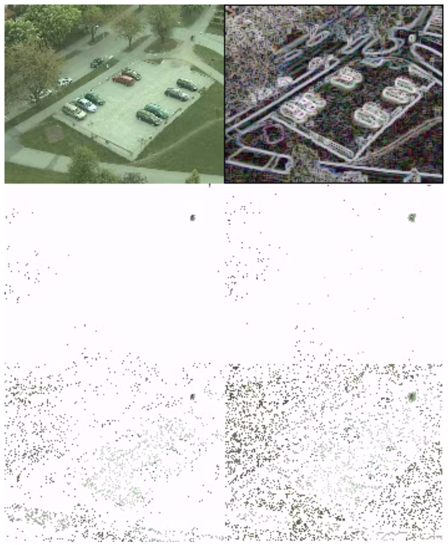
On the applied side:

- Possible ways to speed up the algorithm.
- Object tracking using output from the algorithm.

On the statistical side:

- Utilise the spatial dependency.
- Fast methods for selecting the number of components in mixture models.

Segmentation – Video



Segmentation – Second Video

Parameter Tracks