Assessment of Uncertainties in Life Prediction of Fatigue Crack Initiation in Rails – Influence of Residual Stresses From Manufacturing

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Outline of presentation

• Introduction and motivation for study
• Finite element models and analyses
• Fatigue analysis
• Statistical analysis
• Conclusions
Introduction and motivation for study

- Modern rails are subject to a complex loading situation with high local stresses at the railhead during a wheel passage superposed with a global bending stress in the rail cross section.
  - Hence, a material point near the rail’s surface is subject to cyclic, fatigue loading with rotating principal stresses.

- The manufacturing of rails may give rise to additional concern for fatigue cracks starting from defects in the weld zone.
  - Failures at rail welds and growth of cracks starting in the weld zone have been studied in Mutton and Alvarez [2] and Beretta et al. [3].

- A completed weld, a flash butt weld or a thermite weld, typically exhibits high tensile residual stresses in the web region.
  - These stresses may increase the risk for fatigue failure as they are relatively unaffected by the subsequent resulting high local stresses at the railhead during wheel passages; see Skyttebol et al. [5].
Introduction and motivation for study

S. Marich (1983)

D.E. Sonon (1980)


Introduction and motivation for study

• The influence of the tensile residual stress level (relative to the service stress level) with respect to the risk for initiation of fatigue cracks in the web in the weld zone of a rail is studied.

• Parameter variation in FE analysis:
  – the welding residual stress distribution (shape) and magnitude,
  – the service load magnitude, and
  – the material parameters used in the fatigue life estimation.

• Fatigue analysis:
  – study of fatigue crack initiation using the Dang Van criterion.

• Statistical uncertainty analysis:
  – study of variances according to the Gauss approximation formula.
Finite element models and analyses

- Ringsberg et al. [9]: FE tool developed for the analysis of RCF of rails.
  - **Track model**: track dynamics.
  - **Rail model**: local/detailed RCF analysis.

- The rail model is a 3D FE model made of 8-node brick elements.
  - *Elasto-plastic material behaviour of the steel grade 900A was modelled by a linear kinematic hardening model.*
  - *A Hertzian contact load distribution simulates a traveling wheel (normal and tangential loads).*
  - *An initial welding residual stress field representing a flash butt weld was introduced at the weld position.*
Finite element models and analyses

- Welding residual stress field:
  - Equally large stress components in the vertical (y) and longitudinal (z) directions.
  - No stress in the lateral (x) direction.

- The residual stress components were given a piece-wise linear shape:
  - Tensile magnitude $\sigma_A$ in the web, and the extent of the tensile zone A.
  - This shape is a simplification of the stress fields determined numerically and experimentally by Skyttebol and Josefson [4] and by Tawfik [11].
Finite element models and analyses

- Simulated service load situation:
  - *The heavy haul iron-ore line situated in the North of Sweden (“Malmbanan”).*
  - *10 wheel passages on the rail were simulated (elastic shakedown in the web).*

<table>
<thead>
<tr>
<th>Axle load or $p$ @ velocity (10³ kg or MPa @ km/h)</th>
<th>$\sigma_A$ (MPa)</th>
<th>$A$ (mm)</th>
<th>$\tau_e$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 or 720 @ 70</td>
<td>300</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>22.5 or 1280 @ 100</td>
<td>450</td>
<td>70</td>
<td>168</td>
</tr>
<tr>
<td>25 or 1320 @ 50</td>
<td>600</td>
<td>80</td>
<td>190</td>
</tr>
<tr>
<td>30 or 1410 @ 60</td>
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</tbody>
</table>
Fatigue analysis

- The Dang Van criterion was used for evaluation of the results from the FE analyses.

\[ \tau_a(t) + a_{DV}\sigma_h(t) > \tau_e \]

- \(\tau_a(t)\) is the shear stress “amplitude”,
  - *i.e. deviation from mid value during a stress cycle on a shear plane.*

- \(\sigma_h(t)\) is the **total hydrostatic stress** at elastic shakedown.
  - *It includes the history (residual stresses) from the welding, the global load and the contact load from the wheel passage.*

- \(\tau_e\) and \(a_{DV}\) are material parameters obtained from two fatigue limit tests.
Fatigue analysis

• For every material point considered in the fatigue analysis, a closed material response cycle (MRC) represented by $\tau_a(t)$ and $\sigma_h(t)$ is plotted in the Dang Van diagram.

• Example in the figure:
  – Axle load $30 \cdot 10^3$ kg.
  – Train speed 60 km/h.
  – Fatigue-critical point in the rail web region 100 mm from the rail foot.
Fatigue analysis

INTRODUCTION

FE ANALYSIS

FATIGUE ANALYSIS

STATISTICAL ANALYSIS

CONCLUSIONS
Statistical analysis

- The accuracy in the fatigue crack initiation prediction depends on the uncertainties in:
  - the residual stress shape and magnitude,
  - the service load,
  - the mechanical properties, and
  - the model errors.

- The uncertainty was estimated here considering uncertainties in:
  - the residual stress shape and magnitude,
  - the service load level (the maximum contact pressure), and
  - the fatigue limit.

- The expectancies $E[\cdot]$ and variances $\text{Var}(\cdot)$ of the stochastic variables were obtained from references in the literature:
  - $E[A] \approx 60 \text{ mm}$ and $\text{Var}(A) \approx 302$.
  - $E[\sigma_A] \approx 360 \text{ MPa}$ and $\text{Var}(\sigma_A) \approx 1002$.
  - $E[p] \approx 1410 \text{ MPa}$ and $\text{Var}(p) \approx 2712$.
  - $E[\tau_e] \approx 168 \text{ MPa}$ and $\text{Var}(\tau_e) \approx 252$. 
Statistical analysis

• Assume that $\sigma_A$, $A$, $p$ and $\tau_e$ are stochastic variables one has for the difference $\xi$,

$$\xi = \log(\tau_e) - \log(\tau_a(t) + a_{DV}\sigma_h(t))$$

$$\operatorname{Var}(\xi) \approx \sum_{i=1}^{4} \left( \frac{\partial \xi}{\partial \log(X_i)} \right)^2 \operatorname{Var}(\log(X_i)) + 2 \sum_{i<j} \frac{\partial \xi}{\partial \log(X_i)} \frac{\partial \xi}{\partial \log(X_j)} \operatorname{Cov}(\log(X_i),\log(X_j))$$

• The Gauss approximation formula gives:

$$\operatorname{Var}(\log(X_i)) = \frac{\operatorname{Var}(X_i)}{E[X_i]^2}$$

• Requirement of a zero net longitudinal residual stress component in a vertical plane at the weld. The relation between the variables can be expressed as $\sigma_A \cdot A \approx \text{constant}$ which gives:

$$\operatorname{Cov}(\log(\sigma_A),\log(A)) = -\operatorname{Var}(\log(\sigma_A))$$
Statistical analysis

\[ \tau_a(t) + a_{DV} \sigma_h(t) > \tau_c \]

\[ \xi = \log(\tau_e) - \log(\tau_a(t) + a_{DV} \sigma_h(t)) \]

\[ \text{Var}(\xi) \approx \sum_{i=1}^{4} \left( \frac{\partial \xi}{\partial \log(X_i)} \right)^2 \text{Var}(\log(X_i)) + 2 \sum_{i<j}^{4} \frac{\partial \xi}{\partial \log(X_i)} \frac{\partial \xi}{\partial \log(X_j)} \text{Cov}(\log(X_i), \log(X_j)) \]

<table>
<thead>
<tr>
<th></th>
<th>Position 1: ( d_y = 53 \text{ mm} )</th>
<th>Position 2: ( d_y = 76 \text{ mm} )</th>
<th>Position 3: ( d_y = 100 \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_A ): residual stress level</td>
<td>761</td>
<td>646</td>
<td>749</td>
</tr>
<tr>
<td>( A ): extent of tensile residual stress</td>
<td>1121</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>( \tau_e ): fatigue parameter</td>
<td>625</td>
<td>625</td>
<td>625</td>
</tr>
<tr>
<td>( p ): (maximum) contact pressure</td>
<td>112159</td>
<td>171179</td>
<td>280363</td>
</tr>
<tr>
<td>The “covariance term”</td>
<td>-1027</td>
<td>-137</td>
<td>-183</td>
</tr>
<tr>
<td>\text{Var}(\xi)</td>
<td>113640</td>
<td>172336</td>
<td>281590</td>
</tr>
</tbody>
</table>

The uncertainty in the risk for initiation of fatigue cracks is dominated by the uncertainty in the contact load level.
Conclusions

• The representation of the welding residual stress field was simplified compared with similar investigations.
  – *It was deemed satisfactory for a quantitative investigation.*

• The fatigue analysis showed that:
  – *the presence of welding residual stresses increases the risk for fatigue crack initiation, and*
  – *the higher the magnitude of the stresses ($\sigma_A$) the larger is this risk for fatigue failure.*

• The statistical analysis showed that:
  – *the contact load, $p$, had the greatest influence on $\text{Var}(\xi)$,*
  – *the welding residual stress magnitude, $\sigma_A$, was the second most influencing parameter,*
  – *the fatigue parameter, $\tau_e$, was the third most influencing parameter,*
  – *followed by the welding residual stress distribution, $A$.*