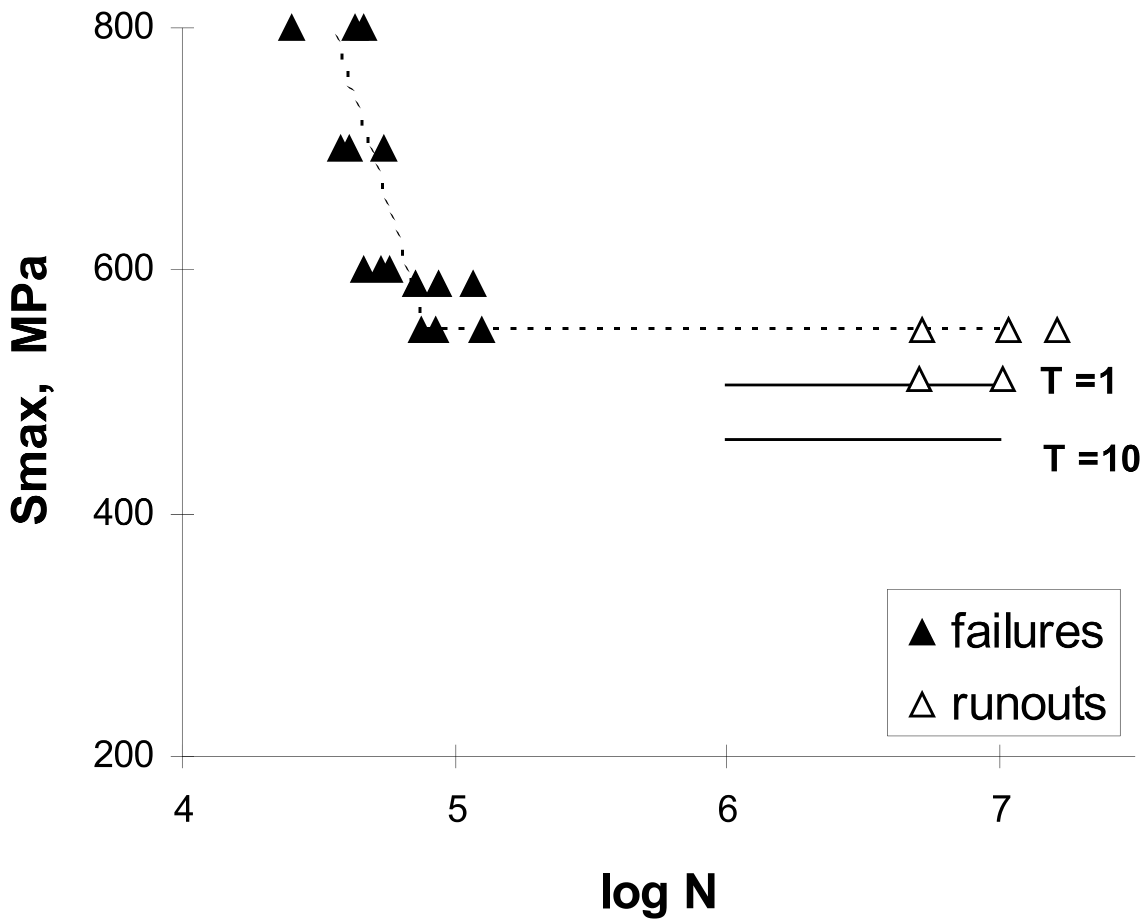
POLITECNICO DI MILANO

A weakest-link analysis for fatigue strength of components containing defects

S. Beretta, E. Soffiati
Dipartimento di Meccanica
Politecnico di Milano

G. Chai
SANDVIK Materials Technology

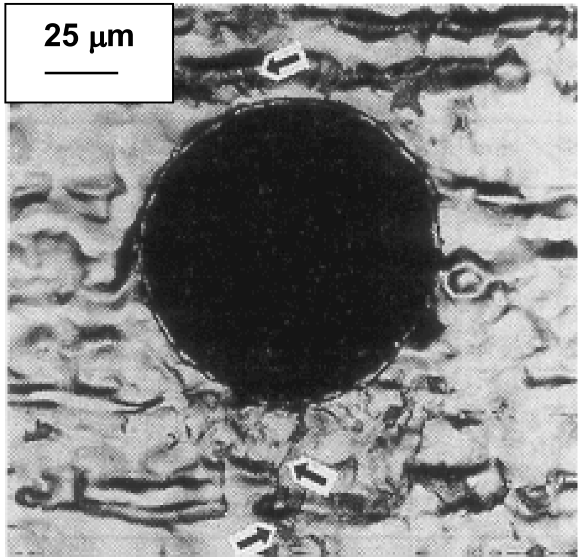
What is the fatigue limit ?



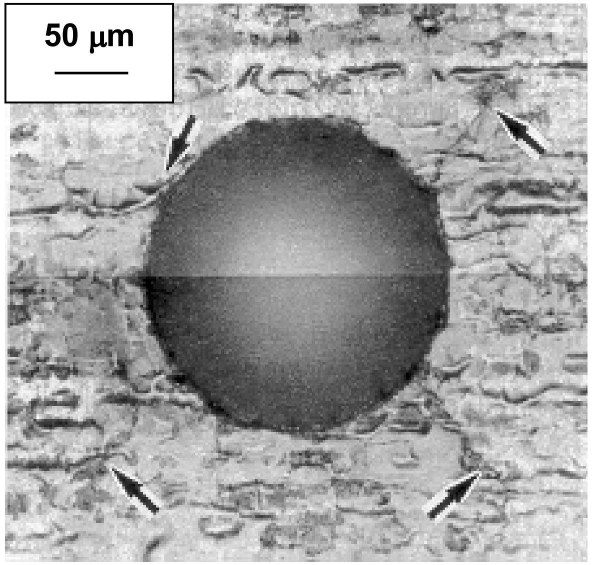
Defects and fatigue strength

Murakami's experiments on specimens containing microholes

Axial loading (bending)



torsional loading (biaxial)

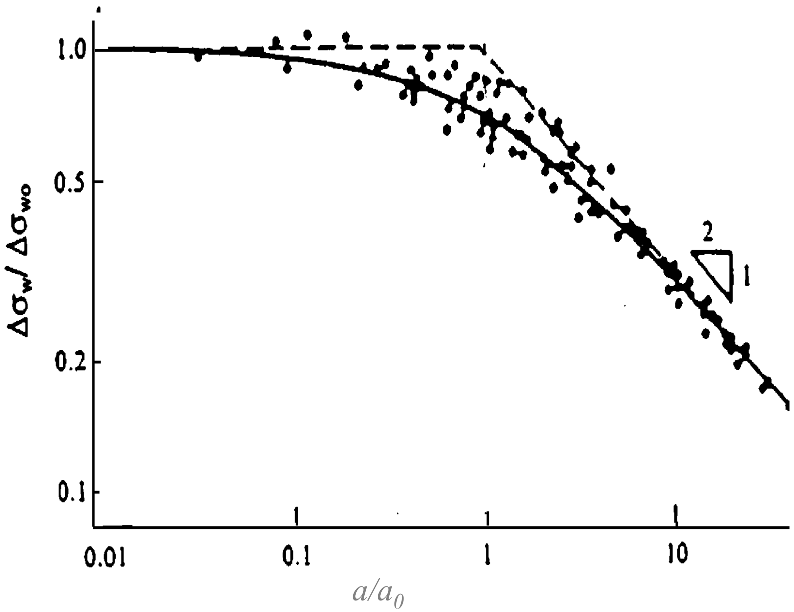


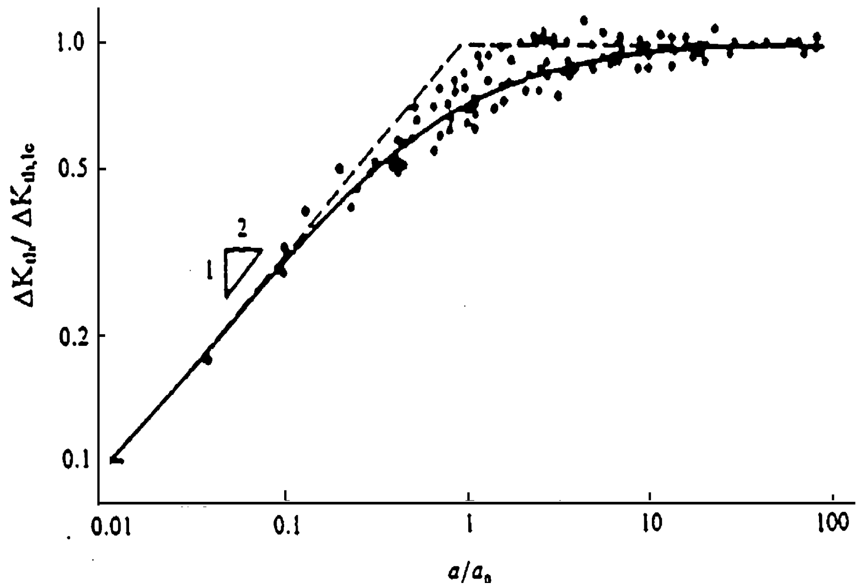
Fatigue limit is the **'non-propagation' condition** for small cracks emanating from the defects

Short cracks and defects

Murakami's idea:

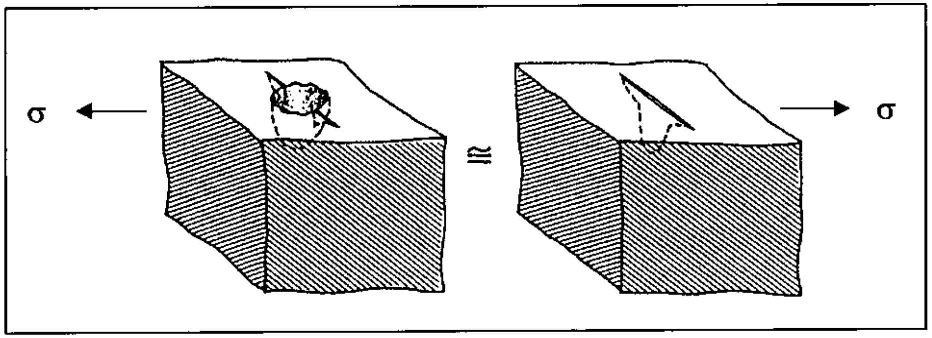
- defects can be treated as short cracks;





threshold ΔK e Δs_{lim} depend on the crack dimension

√area model (Murakami & Endo)



Murakami's equation:

$$\Delta K_I = 0.65 \cdot \Delta\sigma \cdot \left(\pi \cdot \sqrt{Area}\right)^{1/2}$$

Thresholds:

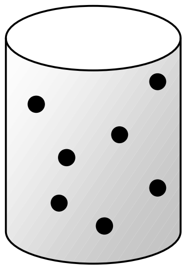
$$\Delta K_{th} = 3.3 \cdot 10^{-3} \cdot (Hv + 120) \cdot \left(\sqrt{area}\right)^{1/3} \cdot (0.5 - R/2)^{0.226 + Hv \cdot 10^{-4}}$$

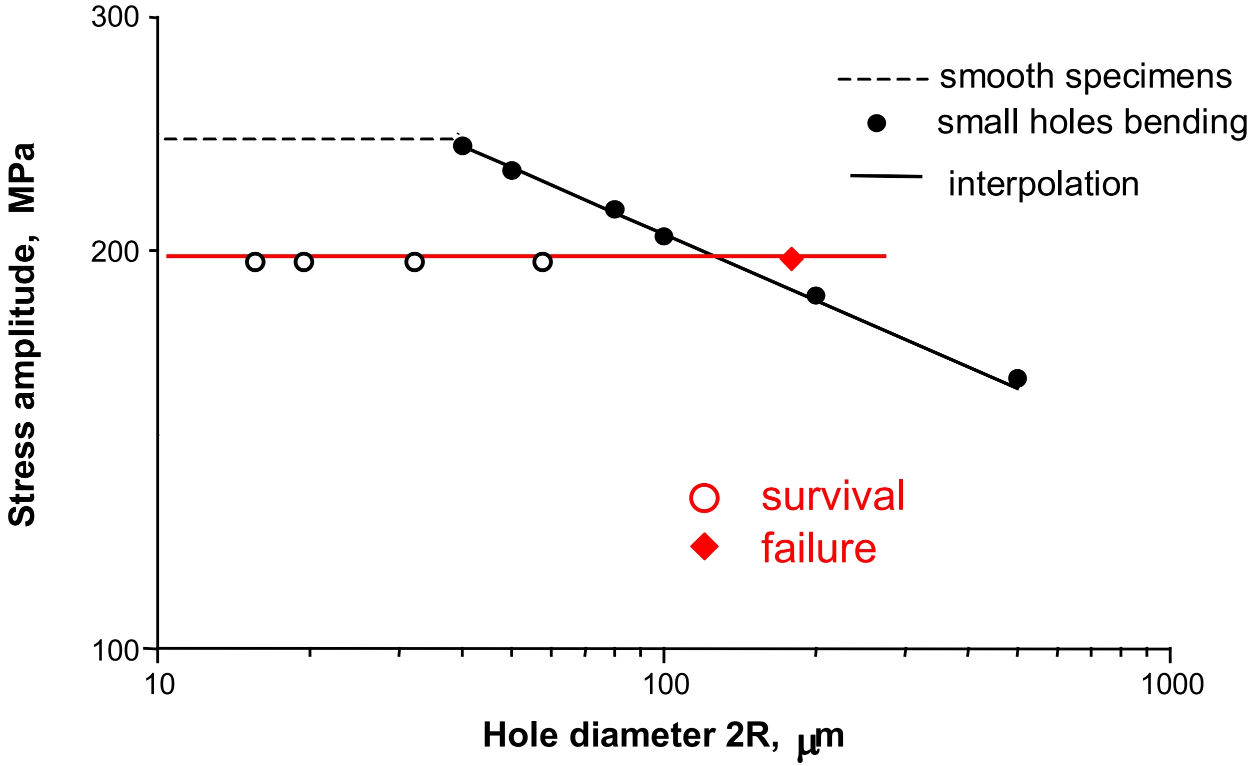
Fatigue limit:

$$\sigma_w = C \cdot \frac{(Hv + 120)}{\sqrt{area}^{1/6}} \cdot (0.5 - R/2)^{0.226 + Hv \cdot 10^{-4}}$$

C = 1.56 internal def.
C = 1.43 surface def.

Extreme defects



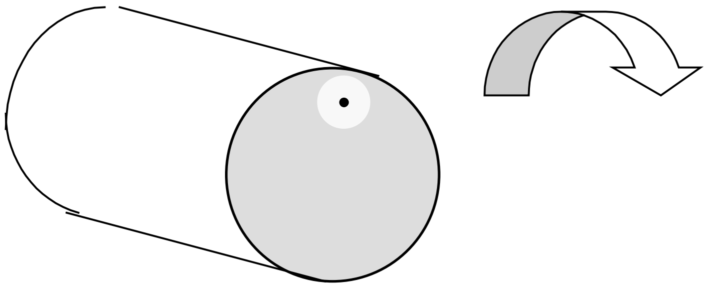


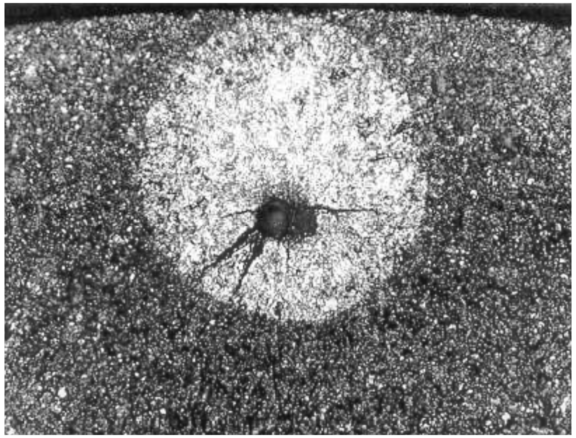
the fatigue is controlled by the **extreme values** of the population of defects **not** by the average dimension

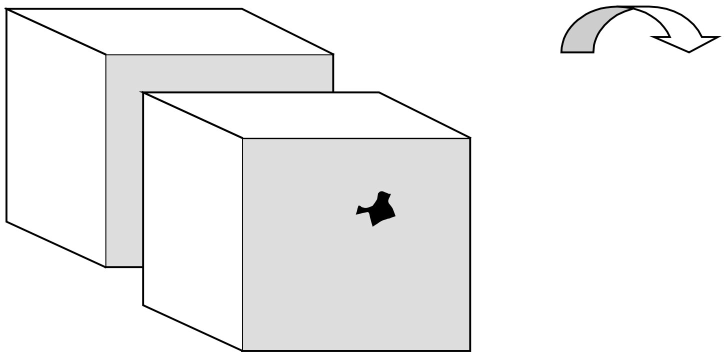
↓

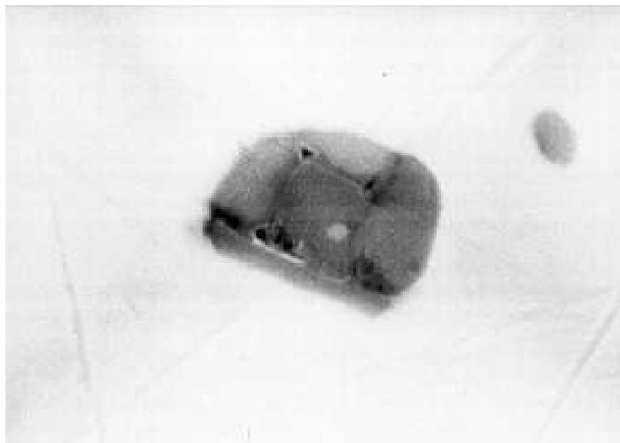
analysis of extremes based on **extreme value sampling**

Extreme defects



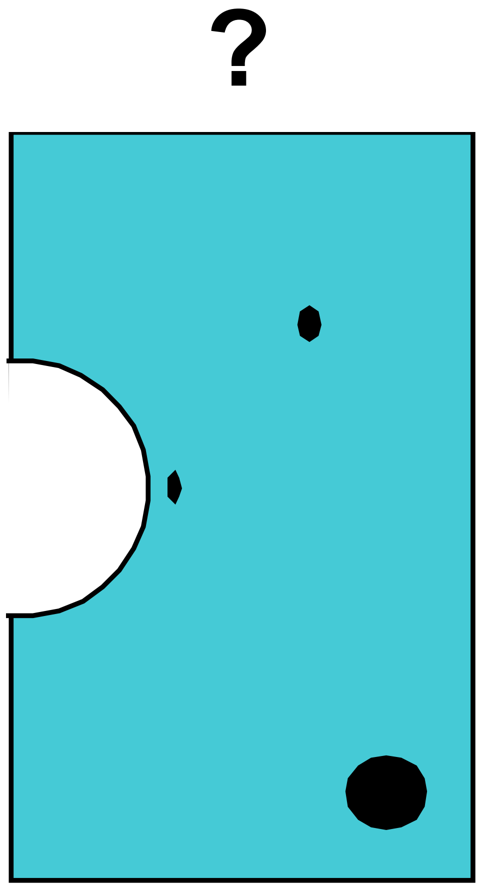






- Methods based on Statistics of Extremes
- new technical recommendations (ESIS e ASTM)

Components with defects



Weakest-link model

⇓

Application to 3 strips of 'super-clean' steels

⇓

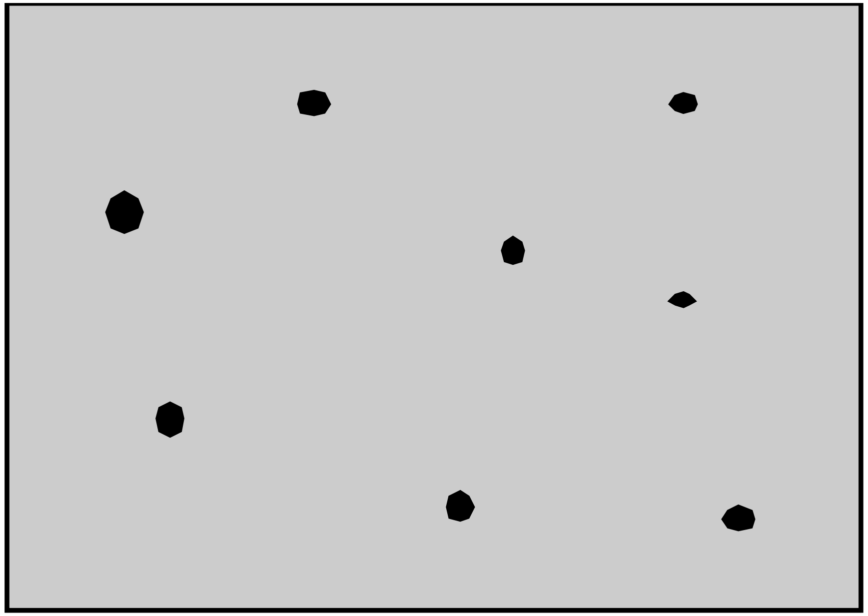
Comparison with fatigue experiments

Extremes

Let us consider ***m* defects** with distribution function F

the distribution function of the maximum defect:

$$F_{\max}(a) = \left[F(a)\right]^m$$



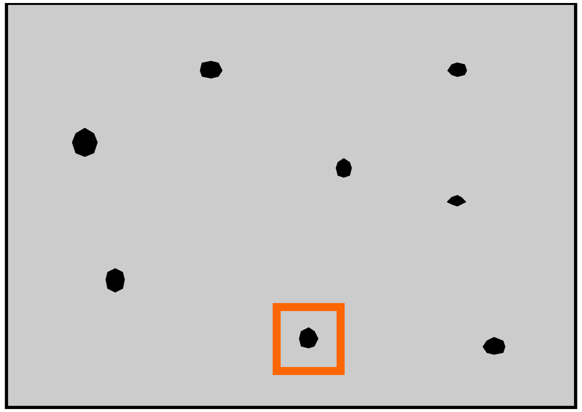
$$m \to \infty$$

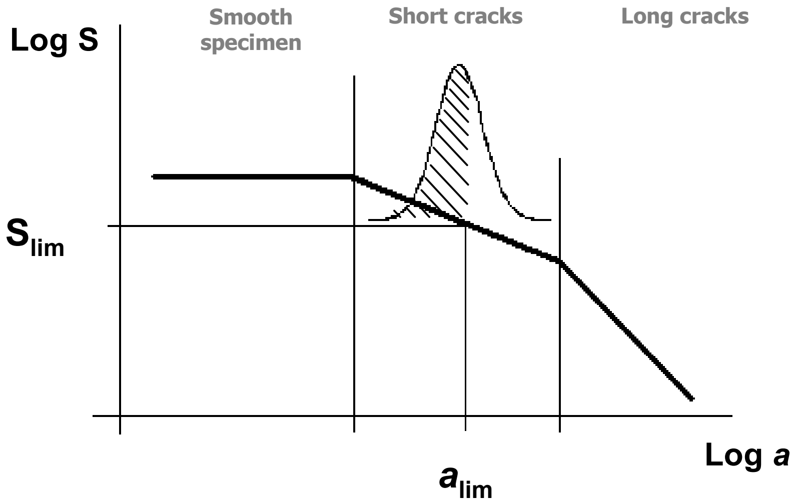
$$F_{\max}(a) \cong G(a, \lambda,\ \delta) = \exp\left(-\exp\left(-\frac{a-\lambda}{\delta}\right)\right)$$

POLITECNICO DI MILANO

Extremes ⇒ Weakest-link

***m* defects**





Weakest-link

$$R_{tot} = R^m$$

$$R = \left\{ \Pr\left(a \le a_{\lim} \right) \right\} = F\left(a_{\lim} \right)$$

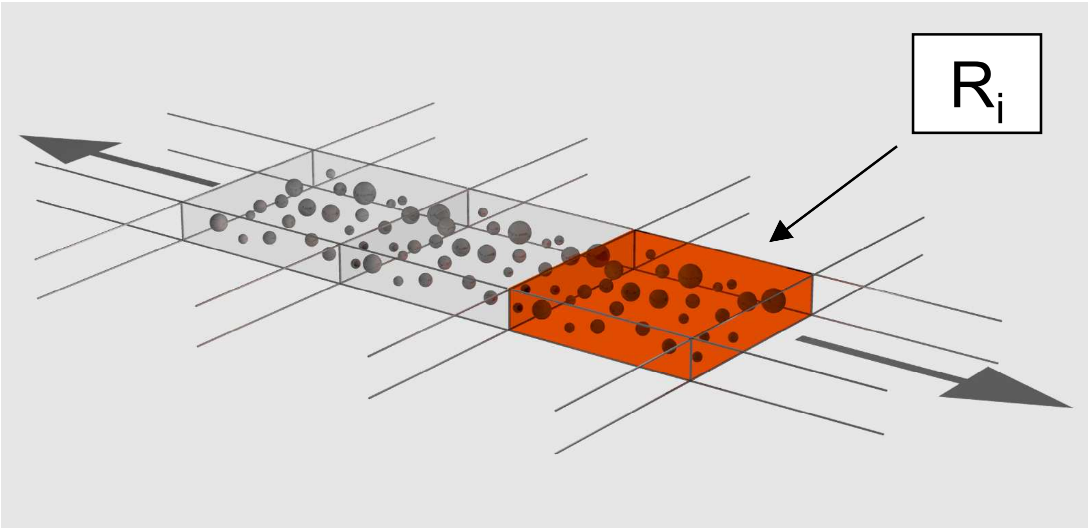
$$R_{tot} = \left[F\left(a_{\lim} \right) \right]^m = \left\{ \Pr\left(a_{\max} \le a_{\lim} \right) \right\}$$

Approaches of WL and Extremes are coincident

POLITECNICO DI MILANO

Weakest-link model

If we imagine the component divided into ***n* domains**:



Weakest-link

$$R_{tot} = \prod R_i \quad \rightarrow \quad R_i = G_V\left(a_{\lim,i}\right)$$

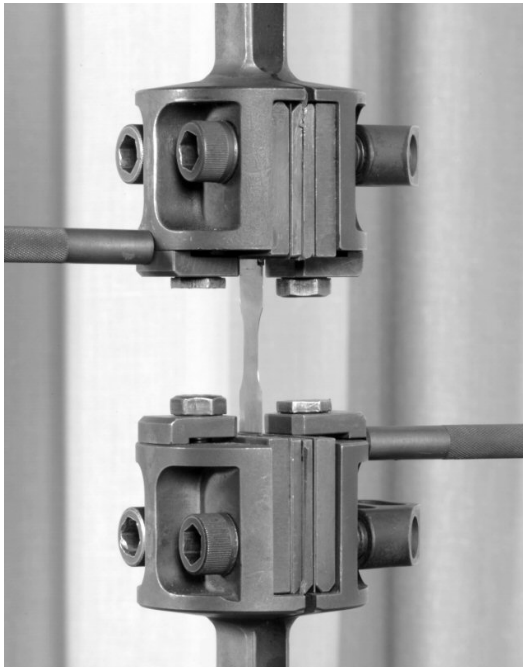
Extremes

$$G_V = \left(G_{V_0}\right)^{\frac{V}{V_0}}$$

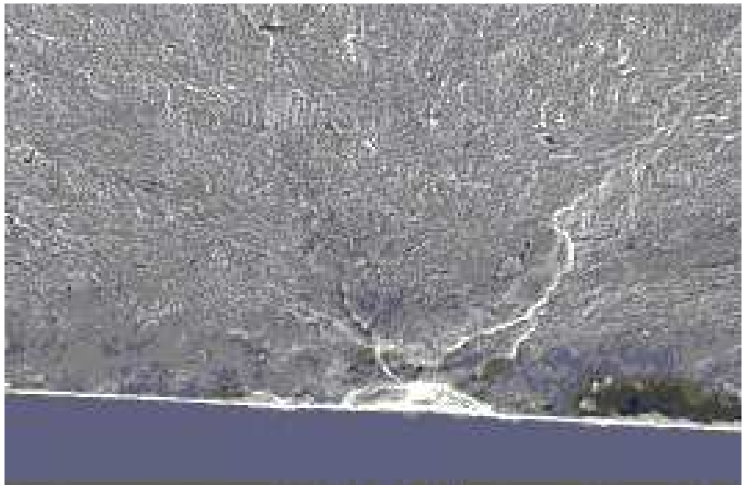
Application

3 series of thin strips of super-clean steels (SANDVIK)

Material	Thickness $[mm]$	HV $[kgf / mm^2]$	Rm $[MPa]$
strip A	0.305	539	1705
strip B	0.305	556	1744
strip C	0.381	581	1649

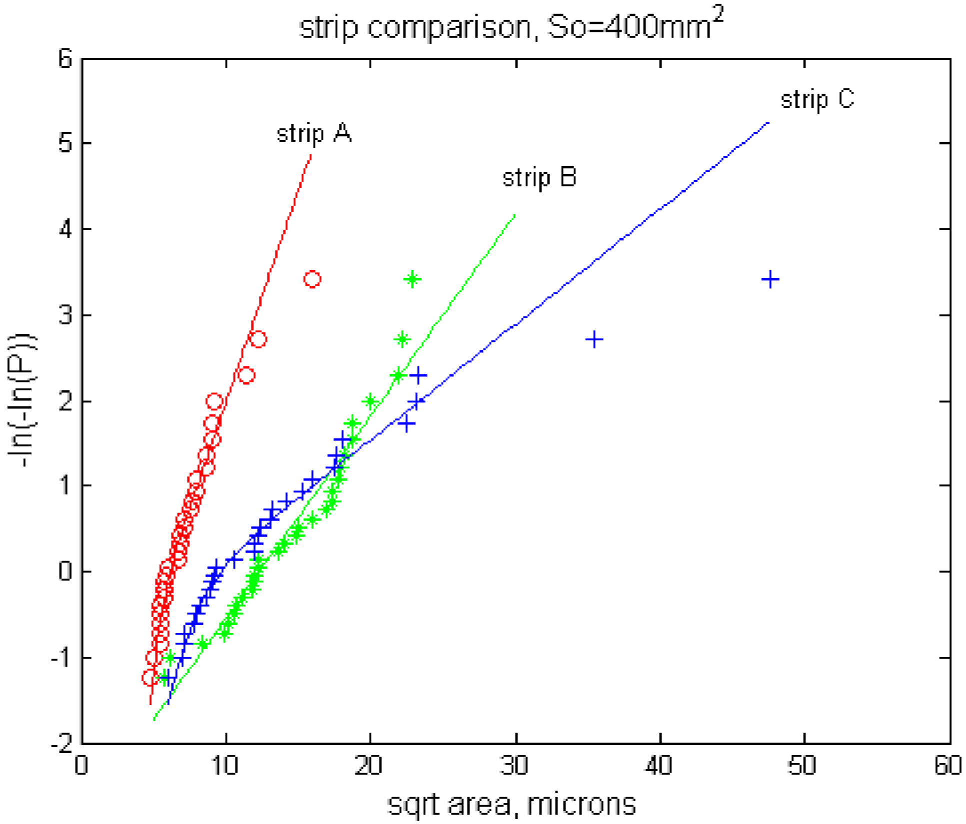


inclusions at fracture origin



Research of extreme defects

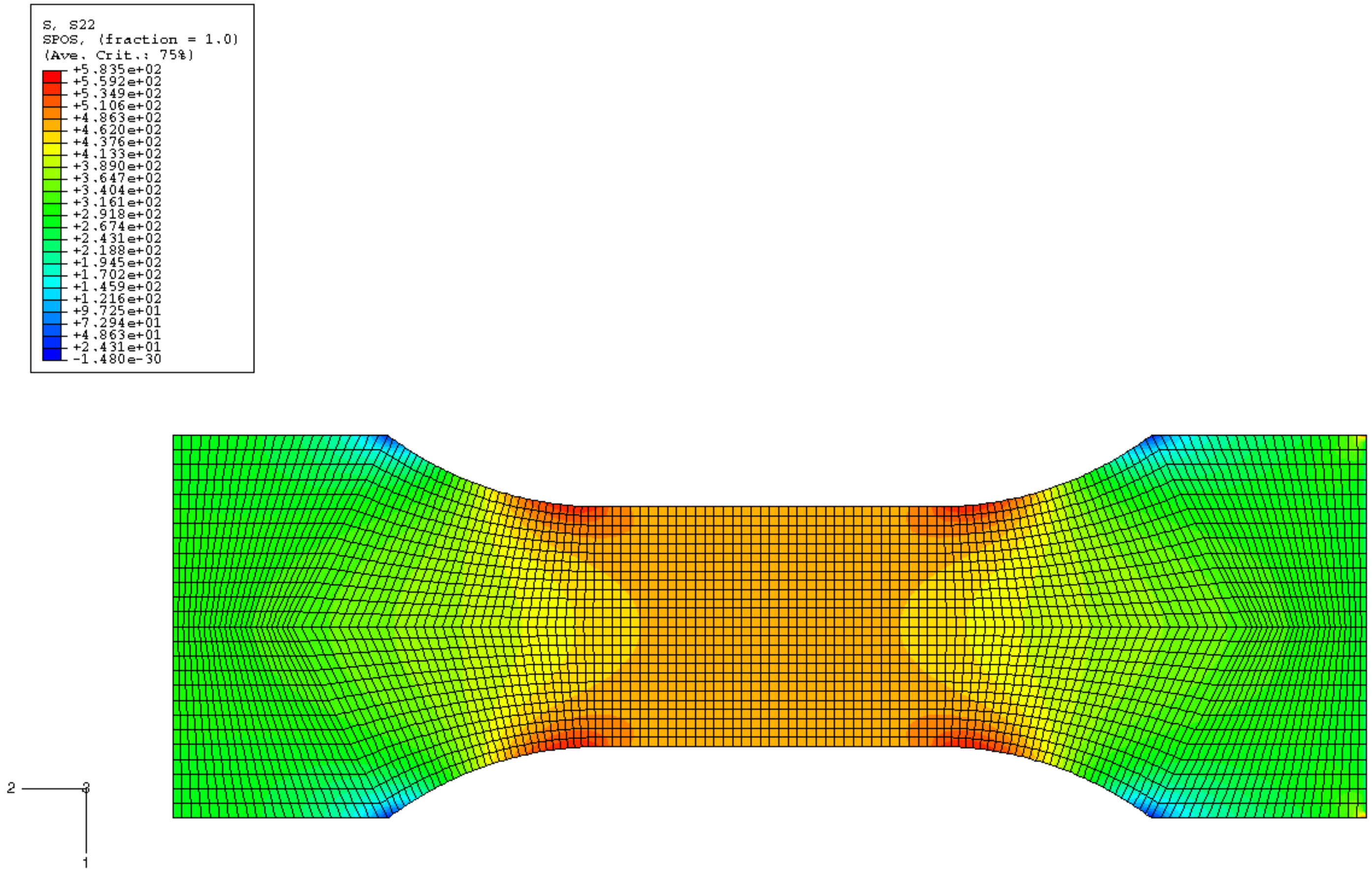
Polished sections



Distribution of maximum defects on S_o = 400 mm^2

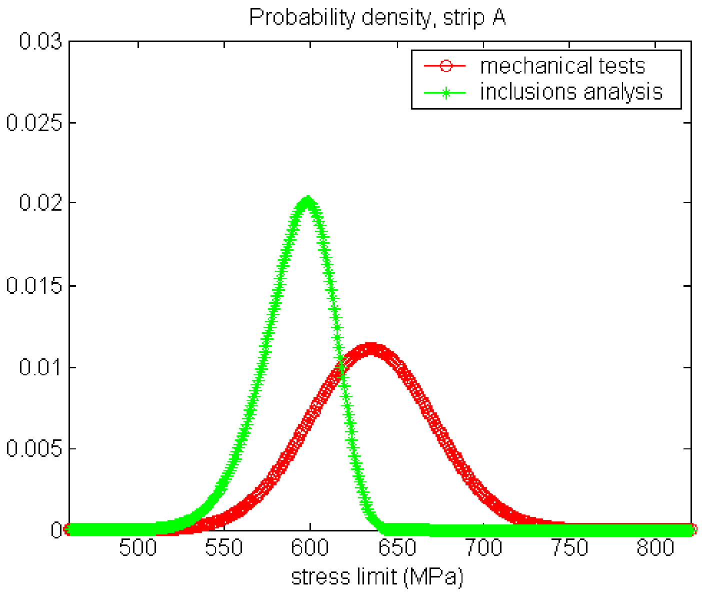
POLITECNICO DI MILANO

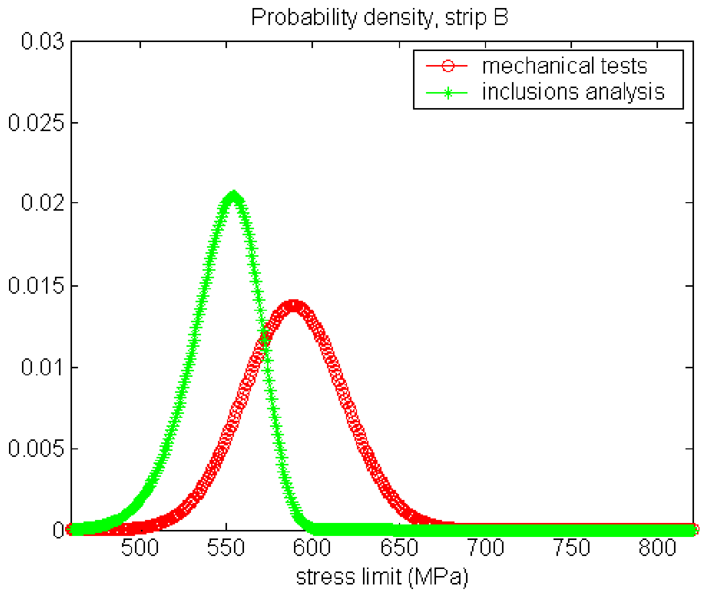
FE analysis of fatigue specimens

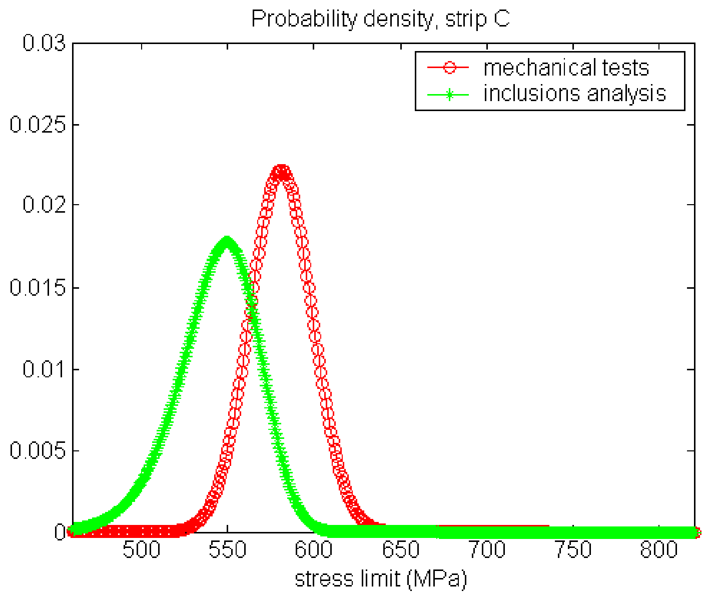


- Calculation of failure probability as a function of S;
- determination of fatigue limit distribution function.

Results



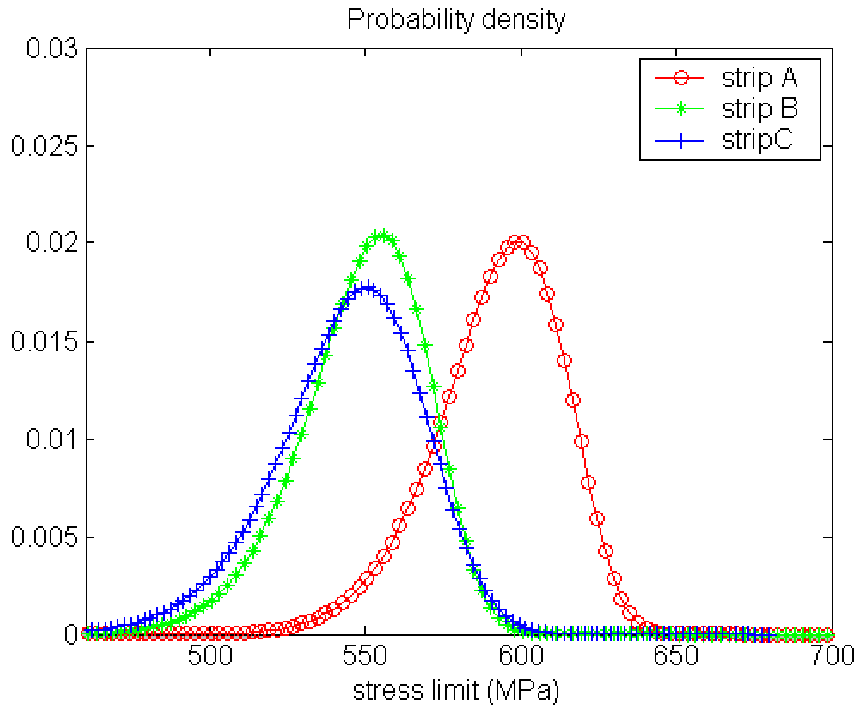




good fatigue strength predictions

POLITECNICO DI MILANO

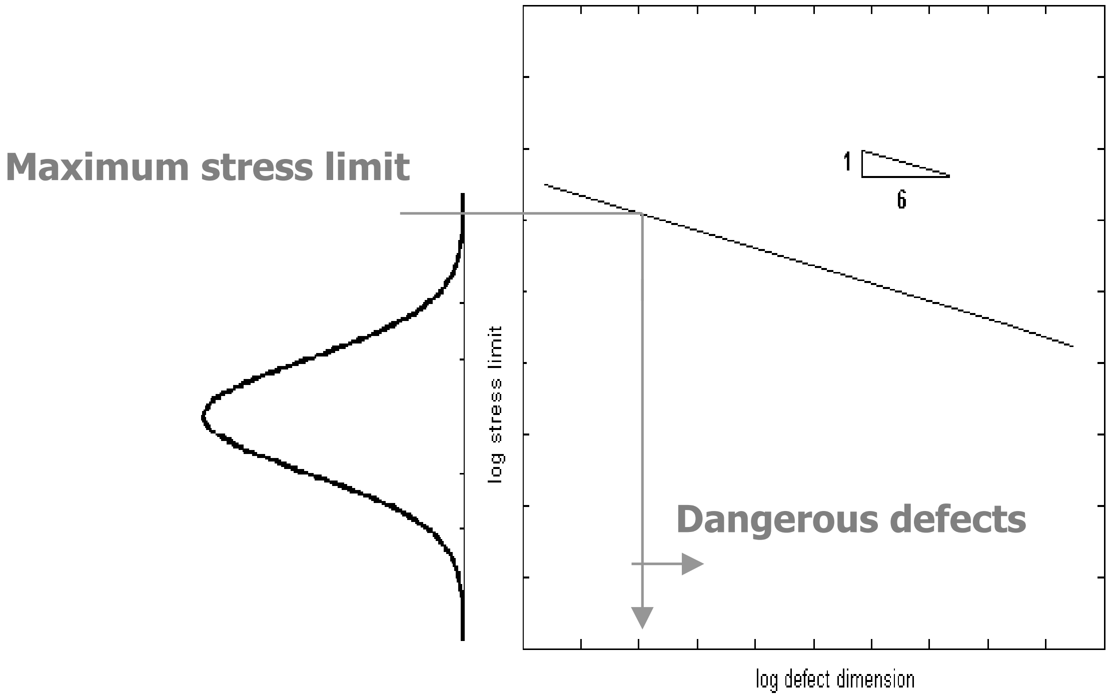
Comparison among strips



strip B has the best hardness → why low fatigue strength ?

extreme defects ?

Comparison among strips



Strip A	Strip B	Strip C	
7	11	11	Critical threshold $[\mu m]$
0.606	1.361	0.722	Critical density / $400\ mm^2$

'density' of detrimental defects

Conclusions

- fatigue limit in presence of defects can be estimated from the Kitagawa diagram of the material under examination

- a Weakest-link model has been proposed in combination with ‘statistics of extremes’ for estimating fatigue strength in mechanical components

- application shows that while for material qualification the maximum defect is a sufficient information, the calculation of the failure probability for a component need also information about defect density