Optimization models for improving periodic maintenance schedules by utilizing opportunities

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Abstract

We present mathematical models for finding optimal opportunistic maintenance schedules for systems, in which components are assigned maximum replacement intervals. Our models are applied to safety-critical components in an aircraft engine, for which maintenance opportunities naturally arise since entire modules are sent to the workshop when maintenance is required on one or more components. Case study results illustrate the advantage of the mathematical models over simpler policies, the benefit of coordinating the maintenance in economically dependent systems, and that our models can be utilized also for strategic investment decision support.

Keywords: Integer Optimization, Opportunistic Maintenance Scheduling, Aircraft Engine Maintenance

Introduction

The importance of performing maintenance operations well—and of improving its state of the art—seems to be impossible to overestimate. According to (Robertson and Jones, 2004), maintenance budgets represent, on average, 20% of the total plant operating budget, ranging from a few percent in light manufacturing to very high percentages in equipmentintensive industry and in the utilities sector. Hidden costs are even more valuable than the ones accounted for in the maintenance budget (Cigolini et al., 2008), as they are related to the leading factors driving the management attention. Maintenance and facilities management practices are also crucial in sustaining safety and eco-efficiency.

A recent study (Forum Vision Instandhaltung, Germany) shows that maintenance costs in the manufacturing industry within the EU amount to roughly $2000 billion per year. Studies over the last 20 years have indicated that around Europe, the direct cost of maintenance is between 4% and 8% of total sales turnover. Also in these cases, it is quite natural to assume that not all the money spent is spent well: according to information gathered by the Swedish Center for Maintenance Management, maintenance is often performed in an un-coordinated and/or corrective only (that is, after failure has occurred) fashion, resulting in a too frequent production shut down; surprisingly often equipment failure is triggered by inspections and the condition monitoring itself. According to a study on fossil power plants (Corio and Costantini, 1989) 56% of the forced outages occured within one week from an intrusive maintenance task. One objective for constructing and studying mathematical models for optimizing the scheduling of maintenance and inspection activities is to mitigate some of these problems, and thereby to contribute to a shift of focus from considering maintenance as mainly a cost-inducing activity to that of an investment in availability.

Related work

The field of maintenance planning is quite vast (Pintelon and Gelders, 1992; Nicolai and Dekker, 2008). One strategy for planning maintenance activities is so called *opportunistic maintenance*, in which a mathematical model is utilized to decide whether, at a (possibly already planned) maintenance occasion, more than the necessary maintenance activities should be performed; we refer to this as *preventive maintenance activities at an opportunity*. The original opportunistic replacement problem was introduced by (Radner and Jorgenson, 1962), who considered a system of stochastically failing components which, upon failure, incur extensive costs for system shut down. When the system is down for whatever reason, components may be replaced at no additional maintenance cost. Thereby, opportunities arise to trade off remaining lives of components in order to avoid maintenance costs associated with future component failure.

Numerous papers address maintenance-scheduling in different industries: power plants (Canto, 2008; Doyle, 2004; Damien et al., 2007); aircrafts and -engines (Almgren et al., 2008; Sarac et al., 2006); production planning (Panagiotidou and Tagaras, 2007). The concepts of *corrective* and *preventive maintenance*, and *maintenance policies* are described in (Wang, 2002). The literature also differentiates between single (e.g., Damien et al., 2007) and multiple item maintenance, and between finite and infinite time horizons.

We consider a multi-component system in an opportunistic setting and assume that a *maximum replacement interval* for each component is given. Our problem is to find a feasible replacement schedule over a finite horizon such that the total maintenance cost is minimized. As stated in (Nicolai and Dekker, 2008), a common assumption in maintenance planning is that of an infinite horizon, and that the development of finite horizon models is deemed essential for maintenance models of multi-component systems to become operational. The origin of a maximum replacement interval can be a policy decision, a safety regulation, or a contract requirement; its interpretation can also be the component's technical life, which is in this article also referred to as *component life* or *life limit*.

Problem description

Our original motivation for studying the replacement problem was to support the operations at the maintenance workshop for jet engines at Volvo Aero Corporation (VAC). An aircraft engine comprises hundreds of parts grouped into modules. To replace a part the corresponding module must first be removed. Some of the parts are safety-critical, meaning that their failure will cause an engine breakdown, possibly with a catastrophic outcome. Therefore, the safety-critical parts are assigned *life limits* (before which the failure probability is effectively zero), before which they must be replaced. (Cf. (George and Lo, 1980; Day and George, 1982) who study nuclear power plants.) All other parts of the engine are replaced after a condition measurement, hence denoted as *on condition*; hence, the maximum interval before their next replacement needs to be estimated. For some of the on condition parts, failure distributions may be computed from historical data and condition monitoring; this information can then be discretized and used as input to optimization models. This was the subject of two PhD projects (Andréasson, 2004; Svensson, 2007). Here, each on condition component is assigned a maximum replacement interval corresponding to its average failure time. Therefore, unscheduled on condition replacements can occur.

The aircraft engine RM12 is composed by seven modules, each comprising a number of on condition and life limited components. Figure 1 shows the module structure in the engine, defining the order of dismantling the modules from the engine. E.g., to free the burner, the afterburner, low pressure turbine (LPT), and high pressure turbine (HPT) must first be removed, in this order. Each of the seven modules comprises several components,

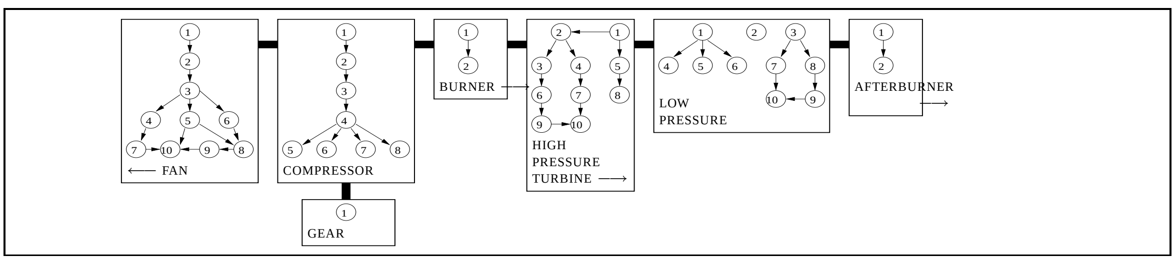

Figure 1 – The RM12 engine is composed by seven modules, each comprising several components.

of which in total 61 are relevant to our analysis; the majority of these possess life limits.

Associated with the engine maintenance are (large) costs for testing, transportation, and administration connected to the maintenance occasions, costs for spare parts (related to their remaining lives), and costs for (dis)assembly and repair work.

Mathematical models

Our replacement optimization model is generic in the sense that it can be used for modelling widely varying applications of maintenance planning, since it utilizes the opportunity for non-periodic maintenance and inspection when appropriate (e.g., (Bohlin et al., 2010; Besnard et al., 2009), which build on our work). We here consider two settings: *(i)* a *single module* with costs related to spare parts and maintenance occasions only, and *(ii)* a *system consisting of several modules* with costs related also to (dis)assembly and repair work.

The single module mathematical optimization model

Consider a set $\mathcal{N}$ of N components (i.e., $N = |\mathcal{N}|$) and a set $\mathcal{T} = \{1, \ldots, T\}$ of time points. A new component $i \in \mathcal{N}$ has a maximum replacement interval of T_i time steps (w.l.o.g., $2 \leq T_i \leq T$). The cost for replacing component i at time $t \in \mathcal{T}$ is $c_{it} \geq 0$ and there is a cost $d_t \geq 0$ associated with a maintenance occasion at time t, independent of the number of components replaced. The objective (1) is to minimize the cost of providing a working system between the time steps 1 and T. We let $z_t = 1$ if maintenance shall occur at time t; $z_t = 0$ otherwise, and $x_{it} = 1$ if component i shall be replaced at time t; $x_{it} = 0$ otherwise. The *opportunistic replacement model* is then defined as that to

$$\underset{(x,z)}{\text{minimize}} \quad \sum_{t \in \mathcal{T}} \left(\sum_{i \in \mathcal{N}} c_{it} x_{it} + d_t z_t \right), \tag{1}$$

$$\text{subject to} \quad \sum_{t=\ell+1}^{\ell+T_i} x_{it} \geq 1, \qquad \ell = 0, \ldots, T - T_i, \quad i \in \mathcal{N}, \tag{2}$$

$$x_{it} \leq z_t, \qquad t \in \mathcal{T}, \quad i \in \mathcal{N}, \tag{3}$$

$$x_{it}, z_t \in \{0, 1\}, \quad t \in \mathcal{T}, \quad i \in \mathcal{N}. \tag{4}$$

The constraints (2) ensure that each component is replaced before the end of its life; (3) enforce the payment of the occasion cost d_t if at least one component is replaced at time t, and—once this cost is paid—induce maintenance opportunities at no extra occasion cost.

The model (1)–(4), developed from the one in (Dickman et al., 1991), is full-dimensional and the constraints are stronger than those in the original model. We have shown (Almgren et al., 2011) that the problem (1)–(4) is NP-hard (Garey and Johnson, 1979), meaning that it cannot be solved in polynomial time in the worst case. For the VAC case it is, nevertheless, efficiently solved by standard integer optimization techniques; this is important due to its extension to stochastic component failures studied in (Patriksson et al., 2012b). In fact, also the whole-engine problem (5)–(11) was solved relatively fast using standard solvers.

Consider an instance of (1)–(4) with $T = 60$, $N = 4$, $T_1 = 13$, $T_2 = 19$, $T_3 = 34$, $T_4 = 18$, and $c_{1t} = 80$, $c_{2t} = 185$, $c_{3t} = 160$, and $c_{4t} = 125$, $t \in \mathcal{T}$; this data reflects the relations between the lives and costs for the fan module of the RM12. Consider also $d_t \in \{0, 10, 1000\}$, $t \in \mathcal{T}$ ($d_t = 10$ representing a reasonable value). Figure 2 shows optimal maintenance schedules for the three cases. The horizontal axis represents the 60 time steps and each vertical bar a maintenance occasion; a dot at a certain height denotes replacement of the corresponding component. Obviously, opportunistic replacement becomes more

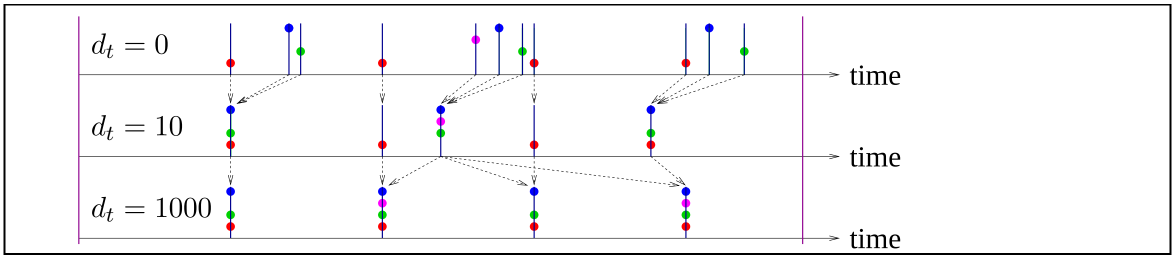

Figure 2 – Optimal maintenance schedules for $d_t = 0$, 10, and 1000 for all t. When d_t increases from 0 to 10 the replacement occasions 1–3, 5–7, and 9–11, are grouped into three occasions. When d_t is increased from 10 to 1000, the last four maintenance occasions are rearranged into three occasions, also resulting in more component replacements.

beneficial with an increasing value of the occasion cost. For $d_t = 0$, the optimal number of replacement occasions is 11 and there is no advantage of replacing a component before the end of its life. Increasing the value of d_t from 0 to 10 decreases the optimal number of replacement occasions from 11 to five; it is now beneficial to replace several components at a time; components are often replaced before their respective life limits are reached. For $d_t = 1000$ it is very important to utilize the opportunity to replace several components at a time; the optimal number of replacement occasions is four (the minimum possible value for this instance), while the number of component replacements is increased.

The several modules' mathematical optimization model

Let $\mathcal{M}$ be the set of modules in the engine and $\mathcal{N}^m$ the set of components in module $m \in \mathcal{M}$. Let $\mathcal{A}$ be the set of all activities necessary to separate the engine into modules, $\mathcal{A}^m$ the set of all activities necessary for releasing module $m \in \mathcal{M}$, $\mathcal{A}(n)$ the set of activities defining the dismantling operations of the modules, and $\delta^m(i)$ the set of components adjacent to component i in module m. Let $c_{it}^m \geq 0$ be the spare part cost of component i in module m at time t, $a_{it}^m \geq 0$ the work-cost of removing component i in module m at time t, $d_t \geq 0$ the occasion cost at time t, $b_{nt} \geq 0$ the cost of activity n at time t, and $T_i^m \geq 2$ the life of a new individual of component i in module m. The binary variables in the model are defined as: $x_{it}^m = 1$ if component i in module m is replaced at time t; $y_{it}^m = 1$ if component i in module m is removed at time t; $w_t = 1$ if the engine is maintained at time t; $v_{nt} = 1$ if activity n is performed at time t; and $z_t^m = 1$ if module m is maintained at time t.

The objective (5) of the several modules' optimization model is to find a replacement schedule that minimizes the total maintenance costs over the planning period, i.e., to

$$\underset{(x,y,z,w,v)}{\text{minimize}} \quad \sum_{t\in\mathcal{T}} \left(\sum_{m\in\mathcal{M}} \left(\sum_{i\in\mathcal{N}^m} \left(c_{it}^m x_{it}^m + a_{it}^m y_{it}^m \right) \right) + d_t w_t + \sum_{n\in\mathcal{A}} b_{nt} v_{nt} \right), \tag{5}$$

$$\text{subject to} \quad \sum_{t=\ell+1}^{\ell+T_i^m} x_{it}^m \geq 1, \qquad \ell = 0, \dots, T - T_i^m,\ i \in \mathcal{N}^m,\ m \in \mathcal{M}, \tag{6}$$

$$x_{it}^m \leq y_{it}^m \leq z_t^m \leq w_t, \qquad t \in \mathcal{T},\ i \in \mathcal{N}^m,\ m \in \mathcal{M}, \tag{7}$$

$$\sum_{j\in\delta^m(i)} y_{jt}^m \geq y_{it}^m, \qquad t \in \mathcal{T},\ i \in \mathcal{N}^m,\ m \in \mathcal{M}, \tag{8}$$

$$\sum_{n\in\mathcal{A}^m} v_{nt} \geq z_t^m, \qquad t \in \mathcal{T},\ m \in \mathcal{M}, \tag{9}$$

$$v_{nt} \leq v_{n't}, \qquad t \in \mathcal{T},\ n' \in \mathcal{A}(n),\ n \in \mathcal{A}, \tag{10}$$

$$x_{it}^m, y_{it}^m, z_t^m, v_{nt}, w_t \in \{0,1\}, \quad t \in \mathcal{T},\ i \in \mathcal{N}^m,\ m \in \mathcal{M},\ n \in \mathcal{A}. \tag{11}$$

The constraints (6) ensure that each component is replaced before the end of its life (cf. (2)); (7) ensure that a component is removed before it is replaced, that only components in modules that are maintained can be removed, and that a module is not maintained unless a maintenance occasion is planned; (8) and (10) ensure that all activities are performed in a feasible order; (9) ensure that module m is separated from the engine prior to maintenance.

Case studies

Previously only simple replacement policies have been used for solving the replacement problem (Hopp and Kuo, 1998). Hence, our first case study compares the single module model (1)–(4) with three such policies for the LPT and HPT of the RM12. The second case study evaluates the current practice of disassembling the modules at the airbase before sending them to the workshop against the possibility to send the whole engine. The third studies the effect of product development on maintenance costs. Data were collected in collaboration with the maintenance team at VAC within a PhD project; some were constructed from engineering judgement (e.g., estimating (dis)assembly times) while others stem from economical data (the RM12 data are confidential and true values are not revealed here).

The policies compared with
The *non-opportunistic maintenance policy* is defined as: "Replace each component only when its maximum replacement interval is reached."

Age replacement policies are classic in the maintenance literature (Barlow and Proschan, 1965, Ch. 3). The age a_i of component i is defined as the time passed since its last replacement. Our *age policy* is defined as: "A maintenance occasion is caused by the maximum replacement interval for some component being reached. Given age limits $\bar{a}_i \leq T_i$, $i \in \mathcal{N}$, component i is replaced at a maintenance occasion if $a_i \geq \bar{a}_i$." Finding optimal values for the age limits $\bar{a}_i$ in an age policy is computationally demanding; we assign $\bar{a}_i := \max\{0, T_i - \delta\}$, where δ is chosen by an exhaustive search (Almgren et al., 2011).

Our value policy resembles the decision process employed at VAC, where it is combined with manual adjustments. The *value policy* is defined as: "A maintenance occasion is triggered if the maximum replacement interval of some component is reached. Each component $i \in \mathcal{N}$ with $c_i > d$ is assigned the value $v_i = c_i \tau_i / T_i$, where τ_i is the time remaining till the maximum replacement interval of component i. An age limit $T_{\min} \leq T$ is given. At a maintenance occasion, component i is replaced if either $c_i > d \geq v_i$ holds or $c_i \leq d$ and $a_i \geq T_{\min}$ hold." The value policy can be interpreted as an age policy with $\bar{a}_i = T_i(1 - d/c_i)$ if $c_i \geq d$ and $\bar{a}_i = T_{\min}$ otherwise.

The single module mathematical model versus simple policies
We compare the single module model (1)–(4) with each of the policies defined above for the LPT and the HPT. The planning horizon T corresponds to 5000 flight hours (fh) and each time step is 50fh. According to the procedure used at VAC, $T_{\min} = 150$fh.

The LPT comprises ten components, of which six are on condition and four are safety critical (life limited). We chose $\delta = 1050$fh. For $d = \bar{d}$ ($\bar{d}$ representing the occasion cost) the total maintenance cost of the schedule obtained by the model (1)–(4) is 34% lower than that obtained from the non-opportunistic policy. Further, as the maintenance occasion cost d increases, the age and value policies and the model (1)–(4) improve compared to the non-opportunistic policy (see Figure 3(a)). Although the number of maintenance occasions resulting from the model (1)–(4) is about one third of that of the non-opportunistic policy, the number of replacements of each component is equivalent for the two methods (see Figure 3(b)). The value and age policies yield fewer maintenance occasions, at the price of more component replacements.

The HPT consists of nine components, of which five are on condition and four are safety critical. We chose $\delta = 250$fh. Figure 4(a) reveals trends for the age policy and the model (1)–(4) similar to those obtained for the LPT. The difference between the costs resulting from the optimization model and the non-opportunistic policy is, however, smaller. For $d = \bar{d}$ the total maintenance cost of the schedule obtained by solving the model (1)–(4) is 9% lower than that of the non-opportunistic policy. The numbers of maintenance occasions are equal for the model (1)–(4) and the age policy, being 40% lower than that of the non-

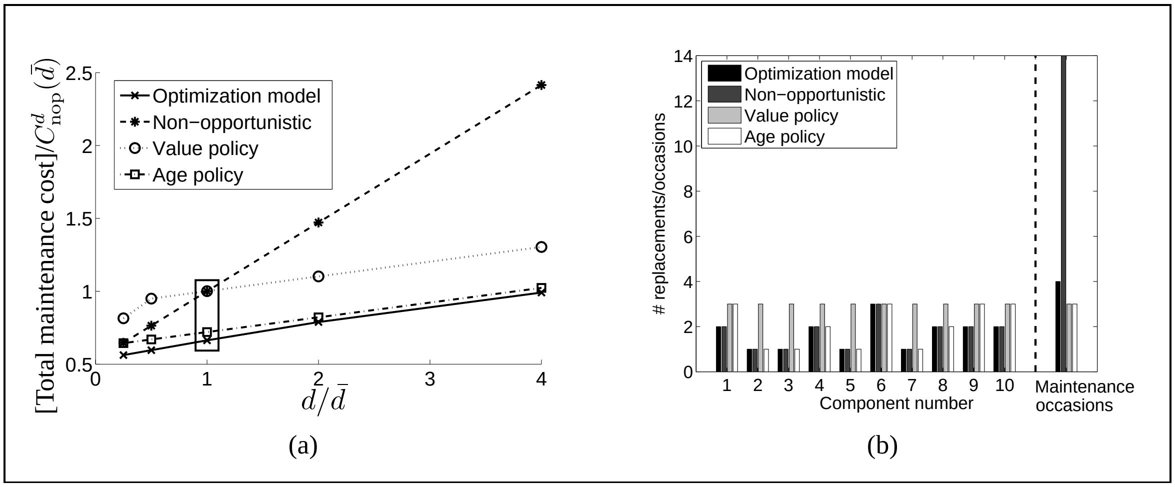

Figure 3 – The LPT instance solved by the model (1)–(4) *and the three policies: (a) Resulting total maintenance costs vs.* $d/\bar{d}$*. The box represents the actual maintenance occasion cost* $\bar{d}$ *at VAC. (b) Numbers of component replacements and maintenance occasions for* $d = \bar{d}$*.*

opportunistic policy. The number of component replacements resulting from the model (1)–(4) and the non-opportunistic and age policies are equal, except for component 2, of which the age policy employs one additional replacement (see Figure 4(b)).

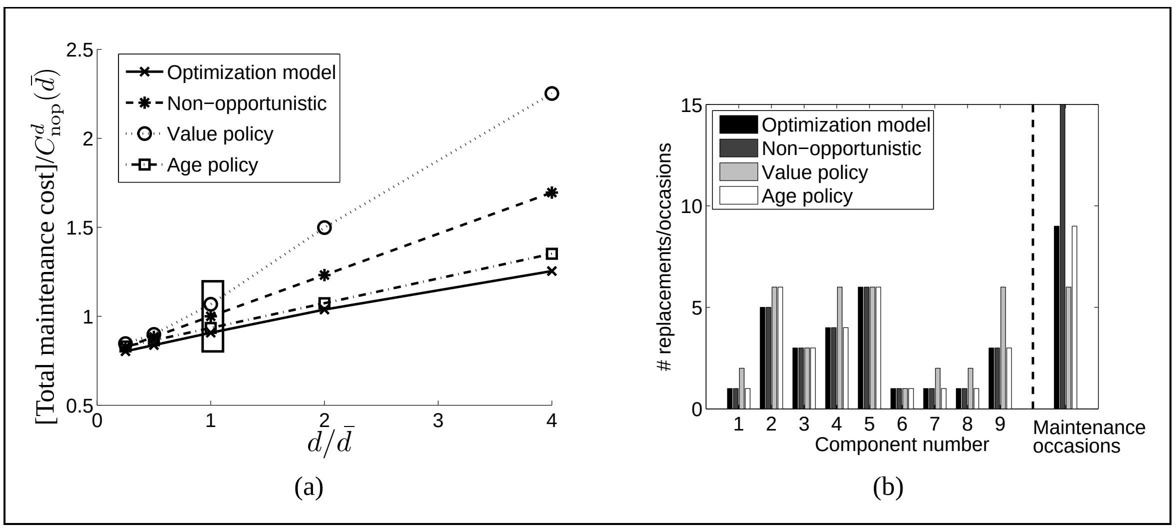

Figure 4 – The HPT instance solved by the model (1)–(4) *and the three policies: (a) Resulting total maintenance costs vs.* $d/\bar{d}$*. The box represents the actual maintenance occasion cost* $\bar{d}$ *at VAC. (b) Numbers of component replacements and maintenance occasions for* $d = \bar{d}$*.*

The benefit of coordinating maintenance activities

We used the model (1)–(4) to plan the maintenance for separate modules and for the whole engine. The realistic planning horizon of 2500fh was discretized into 50 time steps, each representing 50fh; the number of modules/components in the engine was seven/61. When optimizing the schedule for the whole engine, compared to separate modules, the number of maintenance occasions decreased from 15 to 6; the total cost decreased by 12%, and the total number of component replacements increased from 91 to 94. Figure 5 illustrates the corresponding schedules. Cleary, optimizing the maintenance schedule simultaneously for all engine modules yields a better solution than suboptimizing for separate modules.

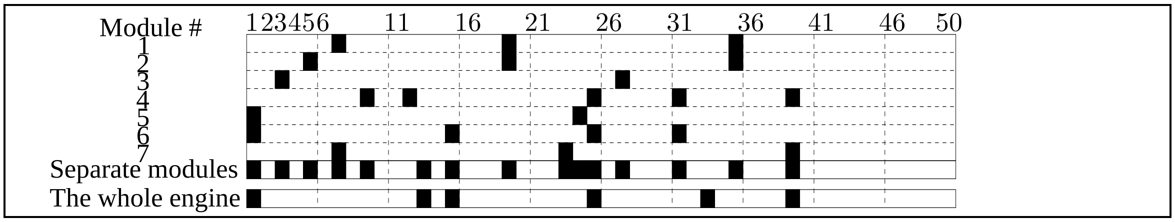

Figure 5 – Maintenance schedules optimized for separate modules and for the whole engine.

The effect on maintenance costs of product development
The optimization of maintenance plans can be used to create a basis for decisions on which product development projects should be carried out. For our application, relevant product development should aim to prolong the lives of individual components. We have made a study which shows that different components possess different potentials for reducing the maintenance costs for an engine. Here, we study the effect of further developing each component such that its respective life is increased to a critical value, at which a reduction of the total maintenance cost occurs. Some components possess lives that are actually longer than the planning period of 2500fh and do not affect the overall costs. The results of the tests are illustrated in Figures 6 and 7. For five out of the 61 components a reduction

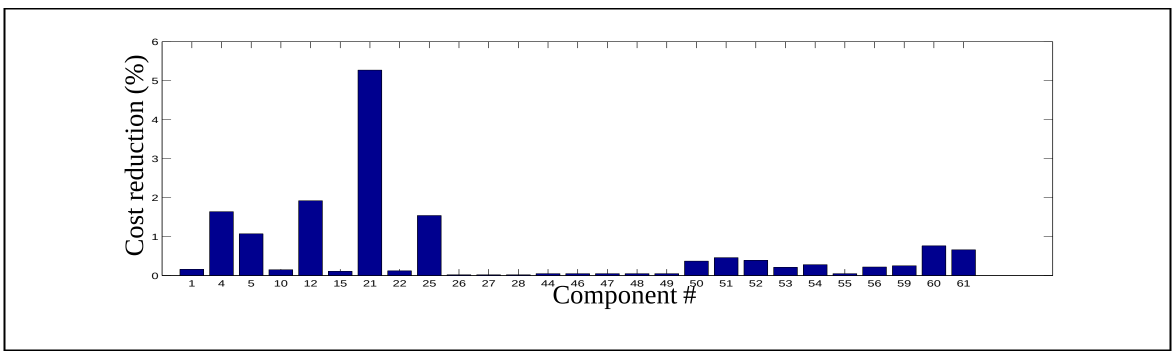

Figure 6 – The potential reduction of the total maintenance costs when a component's life is increased to its critical value. Components excluded possess no potential for cost reduction.

of the total maintenance costs of $> 1\%$ resulted when their respecive quality was improved just as much as needed to influence these costs; for 22 components the corresponding cost reduction was $< 1\%$; for 34 of the components no cost reduction was possible.

Conclusions and further research
Our first case study shows that mathematical optimization can be used to reduce maintenance costs as compared to simpler policies. The second shows a clear benefit of coordinating the maintenance in economically dependent systems and the third shows that models for maintenance decision support can be utilized also for strategic investment decisions.

The research presented in this paper is the basis for several research tracks in maintenance planning optimization. In (Patriksson et al., 2012a, 2012b) replacement decisions

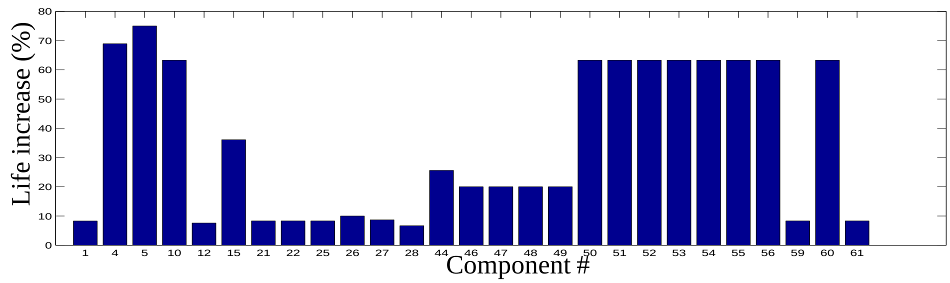

Figure 7 – The critical increase of life for each component. Components excluded possess no potential for reducing the total maintenance costs.

are optimized for components not having a priori assigned maximum replacement intervals, but maintenance decisions are based on the components' current ages.

Work in progress considers bi-objective maintenance scheduling, extending the model (1)–(4), aiming at balacing the costs for maintenance and failure risk. Another track concerns the combined inspection and maintenance planning. We also intend to study models allowing for successive improvements of life distribution estimates through the inclusion of condition monitoring information.

Our modelling methodology contributes to optimization education through its use for assignment work in several undergraduate courses, for illustrating complexity theory, and so far six master/bachelor theses have considered aspects of our models. Our cooperation project with VAC was awarded the Scandinavian First Maintenance Service Award 2010.

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