Numerik der partiellen Differentialgleichungen

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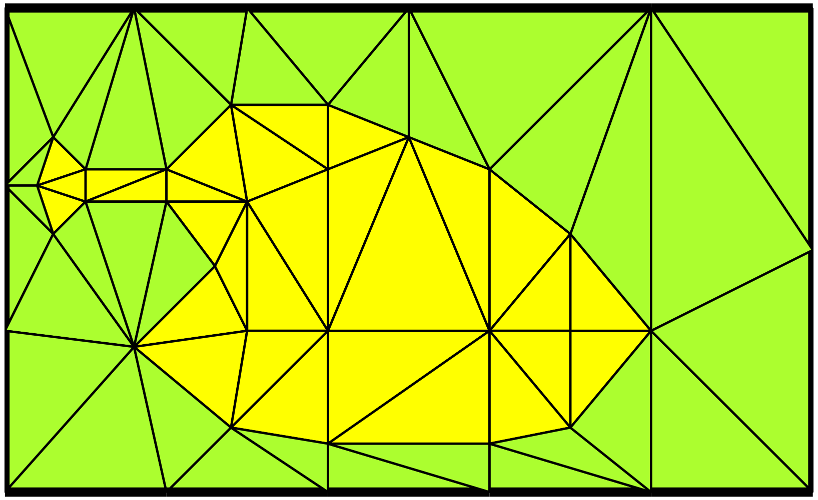


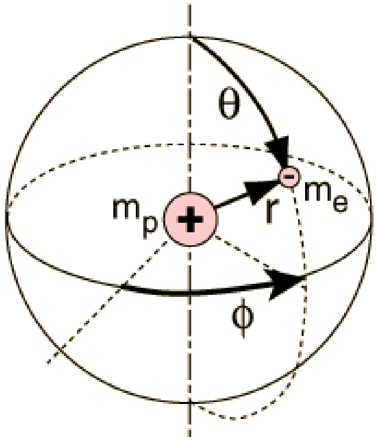
Inhalt

- Moderne Gleichungen
- Beispiele von der klassischen Theorie
- Finite Differenzenverfahren für elliptischen Problemen
- Variationelle Formulierung von partiellen Differentialgleichungen und Finite Elemente Methoden
- Finite Differenzenverfahren für parabolischen und hyperbolischen Problemen

Kapitel I. Moderne Gleichungen

1. Die Schrödinger Gleichung

Erwin Rudolf Josef Alexander Schrödinger $1887 - 1961$ (Vienna).



Bewegung eines Elektron um ein Proton ist durch die Wellenfunktion $u := u(x, y, z, t)$ beschrieben:

$$-i\hbar u_t = \frac{\hbar^2}{2m}\Delta u + \frac{e^2}{r}u,$$

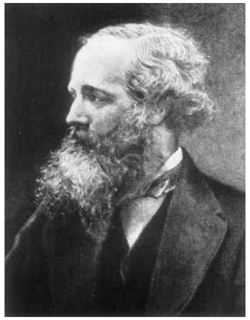
wo $\hbar$ die Plancksche Konstante, m, e Masse, Ladung des Elektrons, und $r = \sqrt{x^2 + y^2 + z^2}$.

Nicht lineare Variante: $\mathrm{i}u_t + \Delta u = \lambda|u|^2 u$.

Anwendung: Schwerewellen im Wasser, Quantenmechanik, Quantenkryptographie, usw.

2. Die Maxwellsche Gleichungen (Vakuum)

James Clerk Maxwell 1831 – 1879 (Edinburgh, UK).



$E := E(x, y, z, t)$ elektrisches Feld ($E : \mathbb{R}^3 \times \mathbb{R} \longrightarrow \mathbb{R}^3$).
$B := B(x, y, z, t)$ magnetisches Feld.

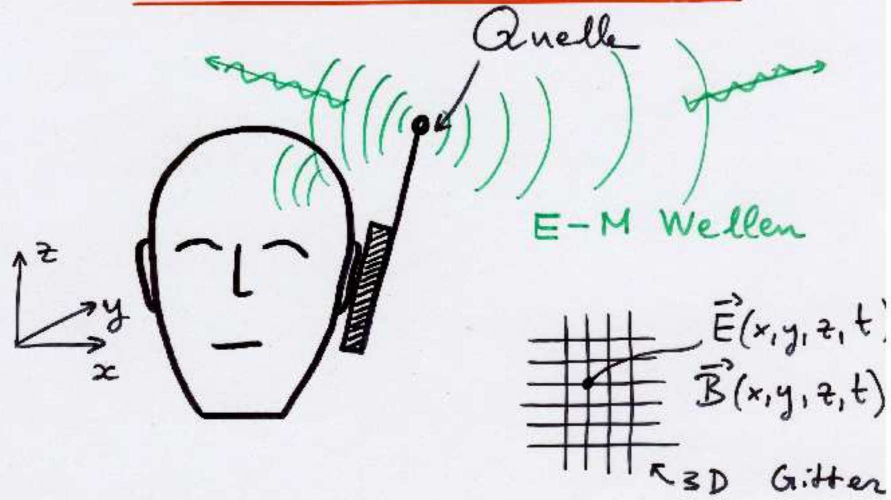
$$\begin{aligned} -\varepsilon \frac{\partial E}{\partial t} + \nabla \times B &= J \\ \mu \frac{\partial B}{\partial t} + \nabla \times E &= 0, \end{aligned}$$

wobei $J := J(x, y, z, t)$ Stromdichte. ε und μ physik. Konstanten.

Anwendung: Radiowellen Ausbreitung, Röntgenstrahlung, usw.

Handy Simulation

HANDY SIMULATION



Maxwell Gleichungen (im Vakuum)

$$
\begin{cases}
\nabla \cdot \vec{E} = \rho / \varepsilon_0 \\
\nabla \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \\
\nabla \cdot \vec{B} = 0 \\
\nabla \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}
\end{cases}
$$

$\vec{E}(x,y,z,t)$ elektrisches Feld

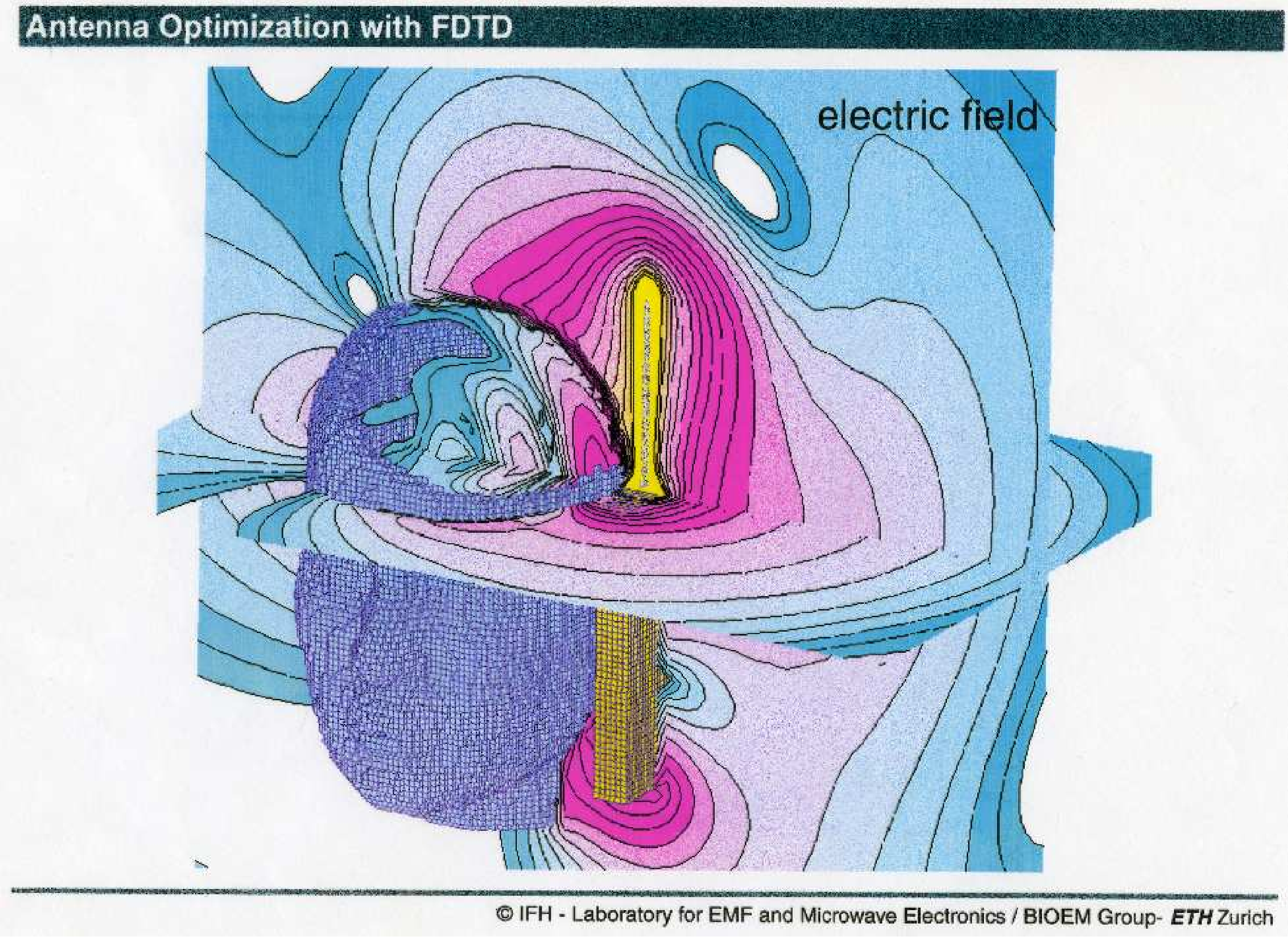
$\vec{B}(x,y,z,t)$ magnetisches Feld

$\rho(x,y,z,t)$ Ladungsdichte

$\vec{J}(x,y,z,t)$ Stromdichte

ε_0, μ_0 physik. Konstanten

Handy Simulation



3. Die Navier-Stokes Gleichung

Claude Louis Marie Henri Navier $1785 - 1836$ (Dijon, France).
George Gabriel Stokes $1819 - 1903$ (Skreen, Ireland).

$u : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ Geschwindigkeit des Fluids. $p : \mathbb{R}^3 \longrightarrow \mathbb{R}$ Druck. ν Viskosität.

$$\begin{aligned} u_t + u \cdot \nabla u + \nabla p &= \nu \Delta u \\ \nabla \cdot u &= 0. \end{aligned}$$

Anwendung: Design des Flugzeuges, Wetterprognose, 1 Million Dollar Problem, usw.

Vom Bombardier

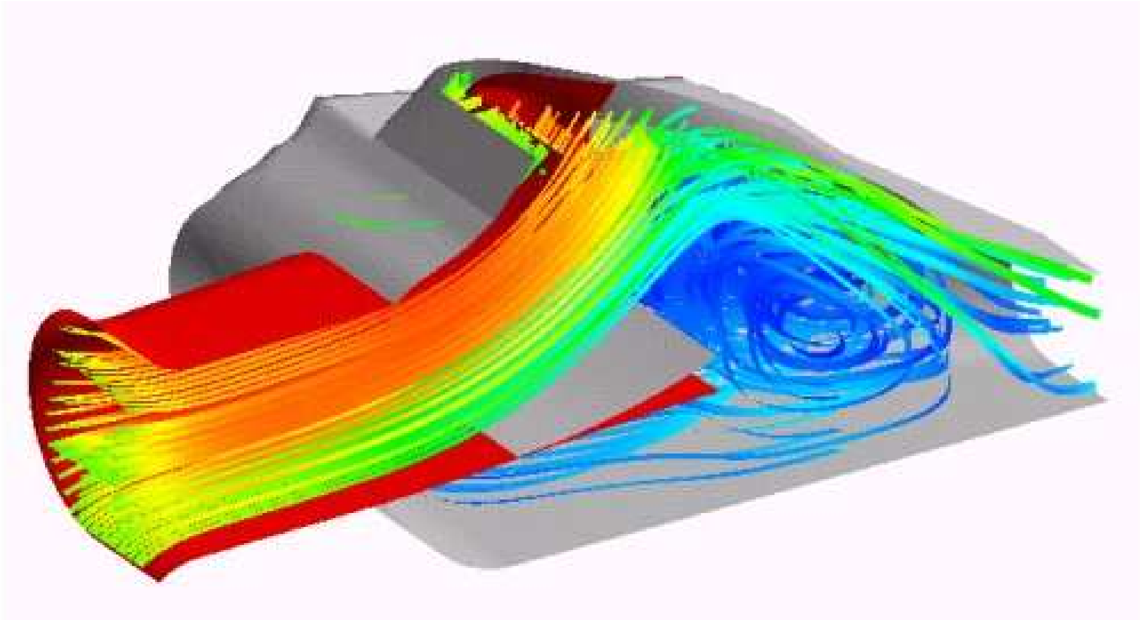


Figure 8: Internal nacelle inlet flow, computed by TASCflow

LANS3D is a 3-0 implicit Navier-Stokes code developed by Fujii and Obayashi. LANS3D employs finite differences and the original code uses a LU-ADI solver and the Baldwin-Lomax algebraic turbulence model. In collaboration with Tohoku University in Sendai, Japan, LANS3D has been extended to include multigrid convergence acceleration, Roe's 3rd order upwind differencing scheme for the convective terms, and the Spalart-Allmaras turbulence model. LANS3D was very useful in assessing the effect of large sideslip angles on the aerodynamic side loads on the Dash 8 fuselage, dorsal fin, and vertical tail. Figure 7 shows a view of the computed flowfield on the leeward side of the Dash 8 at 30 degrees of sideslip. The complex flow pattern shows multiple lines of flow separation and attachment. TASCflow, a general-purpose commercial 3-D Navier-Stokes code developed and marketed by AEA Technology, was acquired in 1996. TASCflow employs finite volume discretization and an implicit multigrid solver. It offers many features including conjugate heat transfer, combustion modeling, and rotating and multiple frames of reference. TASCflow has been used at Bombardier Aerospace Toronto primarily for the analysis of propulsion-related configurations. Figure 8 shows a computed solution of the Dash 8-400 nacelle inlet that includes the complex by-pass duct.

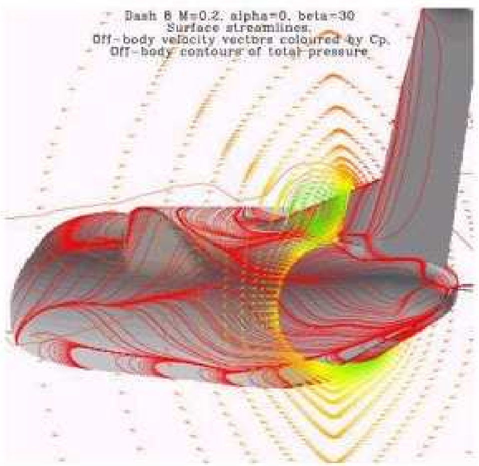


Figure 7: LANS3D computation of the Dash 8 fuselage/fin/dorsal at large sideslip angles