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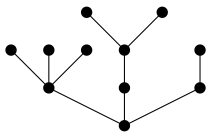
Exercise 4: Construct the order condition, that belongs to the following tree τ.



τ

What is the elementary differential, that belongs to this tree?

Exercise 5:

Compute the adjoint method to the symplectic Euler method.

Exercise 6:

Show, that the Störmer-Verlet method is symmetric.

Exercise 7:

(a) Show that a collocation method is symmetric if and only if $c_i + c_{s+1-i} = 1$ for $i = 1, \ldots, s$.
Hint: Show that the adjoint method is a collocation method with $c_i^* = 1 - c_{s+1-i}$.

(b) Show that the Gauß collocation methods are symmetric.

Programming Exercise 3: *Nyström methods*

Consider the second order differential equation

$$y''(t) = g\big(t, y(t), y'(t)\big)\,, \quad y(t_0) = y_0\,, \quad y'(t_0) = y'_0\,.$$

For real coefficients c_i, $\bar{b}_i$, $\bar{a}_{ij}$, $\hat{b}_i$ and $\hat{a}_{ij}$ a Nyström method for this problem is given by

$$\begin{aligned} l_i &= g(t_0 + c_i h,\ y_0 + c_i h y'_0 + h^2 \sum_{j=1}^{s} \bar{a}_{ij} l_j,\ y'_0 + h \sum_{j=1}^{s} \hat{a}_{ij} l_j)\,, \\ y_1 &= y_0 + h y'_0 + h^2 \sum_{i=1}^{s} \bar{b}_i l_i\,, \\ y'_1 &= y'_0 + h \sum_{i=1}^{s} \hat{b}_i l_i\,. \end{aligned}$$

(a) Show, that the Störmer-Verlet method is a Nyström method applied to the special problem $y''(t) = g\big(t, y(t)\big)$. Compute the coefficients.

(b) *Charged particle in a magnetic field*

Let the vector $b = (b_1, b_2, b_3)^T$ represent the magnetic field. Then, the equations of motion of a charged particle in this field are given by

$$m q''(t) = -\gamma \frac{q}{\|q\|^3} + b \times q'(t)\,,$$

where m is the mass of the particle and γ is a constant.

Set $m = \gamma = 1$, $q_{\text{init}} = (1,1,1)^T$, $q'_{\text{init}} = (0,0,0)^T$ and $b = (0,0,1)^T$. Implement a generalized Störmer-Verlet method with coefficients c_i, $\bar{a}_{ij}$, $\bar{b}_i$ and $\hat{b}_i$ from the first part of the exercise as well as $\hat{a}_{11} = \hat{a}_{21} = 1/2$ and $\hat{a}_{21} = \hat{a}_{22} = 0$ and compute the numerical solution on the time interval $[0, 50]$ with time step size $h = 10^{-2}$. Plot the second component q_2 of the solution versus the first component q_1.

Note: Matlab templates for the programming exercises can be downloaded from `https://na.math.kit.edu/cohen/`.

Discussion in the exercise class on 24.5.2012.