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Exercise 8: Prove the following corollary from the lecture:

A partitioned Runge-Kutta scheme for

$$\begin{cases} \dot{p} = f(p,q) \\ \dot{q} = g(p,q) \end{cases}$$

conserves linear invariants $I(p,q) = d_1^T p + d_2^T q$ with constant vectors d_i, if $b_i = \hat{b}_i$ or if $I(p,q)$ only depends on p or q.

Exercise 9: Prove the following theorem from the lecture:

If the coefficients of a partitioned Runge-Kutta method satisfy

$$b_i \hat{a}_{ij} + \hat{b}_j a_{ji} = b_i \hat{b}_j \,,$$
$$b_i = \hat{b}_i$$

for $i, j = 1, \ldots, s$, then the partitioned Runge-Kutta method conserves quadratic invariants of the form $Q(p,q) = p^T D q$ for an arbitrary (constant) matrix D of the proper dimensions.

Exercise 10:

Show that the symplectic Euler scheme conserves quadratic invariants of the form $Q(p,q) = p^T D q$ for an arbitrary (constant) matrix D of the proper dimensions.

Programming Exercise 4: *Lotka-Volterra Problem*

We consider the following problem

$$\dot{u} = u(v-2) \,,$$
$$\dot{v} = v(1-u) \,.$$

(a) Compute numerical approximations to the exact solution on the interval $[0, 24]$ using the time step size $h = 0.12$ and employing

  (i) the explicit Euler method with starting value $(u(0), v(0)) = (2, 2)$,

  (ii) the implicit Euler method with starting value $(u(0), v(0)) = (4, 8)$,

  (iii) the symplectic Euler method with starting values $(u(0), v(0)) = (4, 2)$ and $(u(0), v(0)) = (6, 2)$.

(b) Plot a phase diagram of the solutions obtained from part (a). Namely plot the second component v versus the first component u. Specifically mark the initial values in the diagram.

(c) Show that $I(u, v) = \ln(u) - u + 2\ln(v) - v$ is an invariant for the above problem. What do you observe for the numerical solutions?

Discussion in the exercise class on 31.5.2012.