Mini-course on Stochastic Differential Equations

Course Description.
The recent years has seen a growing interest in stochastic modeling as stochastic differential equations (SDEs) started to play a more and more important role in divers branches of science and industry. This includes, for instance, applications areas from biology (population growth models), mechanics (Langevin equation), or finance (Cox-Ingersoll-Ross model). Furthermore, as exact solutions to SDEs (basically an ordinary differential equation with the right hand side perturbed by a white noise) are rarely known, one must simulate SDEs numerically. It is therefore essential to understand basic concepts of convergence of numerical methods for SDEs.

The mini-course will provide an accessible introduction to stochastic differential equations and numerical methods to solve them. The focus will be on main ideas and important concepts not on proving results. Furthermore, we will discuss applications from physics, biology, and finance. Prerequisites will be kept to a minimum (basic competence in algebra and calculus is assumed, familiarity with fundamental concepts from probability theory and numerical analysis helps).

Topics covered in the mini-course are:

- Background: probability theory, random variables, and standard Brownian motion
- Ito stochastic integrals
- Stochastic differential equations
- First numerical methods for stochastic differential equations

The mini-course will be complemented by exercises and practical exercises in Matlab.

Background.
First courses in algebra and calculus. Basic knowledge of Matlab (for the practical exercise sessions). Basic knowledge in numerical methods for ordinary differential equations (desirable). Basic knowledge of probability theory (desirable).

Target Audience.
Advanced Bachelor students. Master students. PhD students. Students from physics and other sciences with a basic knowledge in mathematics are more than welcome.

Tentative Schedule.
Background (Two times two hours). Stochastic integration (Two times two hours). Stochastic differential equation (Two times two hours). Numerical schemes (Two times two hours). Possible supervision for the computational tasks.
The course starts on Wednesday the 2th of May 2018 and ends Friday the 11th of May 2018. Two hours lectures per day are planned (this could be modified according to the preferences of the participants).

Main References.
The mini-course is inspired by some parts of the following references:

E. Allen: *Modeling with Itô stochastic differential equations* (introductory text)

L.C. Evans: *An introduction to stochastic differential equations* (lecture notes based on a book with the same name),
https://pdfs.semanticscholar.org/d66c/a1516e1a9a9247f94841ccfbb262cf26d5e4.pdf

D.F. Griffiths and D.J. Higham: *Numerical methods for ordinary differential equations* (some chapters offer a nice introduction to stochastic differential equations),
http://www.springer.com/gp/book/9780857291479

D.J. Higham: *An algorithmic introduction to numerical simulation of stochastic differential equations* (very nice and accessible reference for matlab implementation),
http://dx.doi.org/10.1137/S0036144500378302

P.E. Kloeden and E. Platen: *Numerical Solution of Stochastic Differential Equations* (classic reference on the subject),
http://www.springer.com/us/book/9783540540625

B. Øksendal: *Stochastic Differential Equations: An Introduction with Applications* (classic reference on the subject),
http://th.if.uj.edu.pl/~gudowska/dydaktyka/Oksendal.pdf

Contact.
David Cohen, Umeå University
✉ david.cohen@umu.se
http://snovit.math.umu.se/~david/