

WEEK 1: POPULATIONS DYNAMICS  
Assignments for Mathematical Biology, VT19

---

1. The spruce budworm model is given by

$$\frac{du}{dt} = ru \left( 1 - \frac{u}{q} \right) - \frac{u^2}{1 + u^2}.$$

Find the curve in  $(r, q)$ -space that separates the region of one steady from three steady states.

2. Consider the Lotka-Volterra system

$$\begin{aligned} \frac{dx}{dt} &= ax - bxy \\ \frac{dy}{dt} &= -cy + dxy. \end{aligned}$$

Show that if  $T$  is the period of oscillation then the time averages are given by:

$$\begin{aligned} \frac{1}{T} \int_0^T x(t) dt &= \frac{c}{d} \\ \frac{1}{T} \int_0^T y(t) dt &= \frac{a}{b}. \end{aligned}$$

3. Consider the system

$$\begin{aligned} \frac{dN_1}{dt} &= r_1 N_1 \left( 1 - \frac{N_1}{K_1 + b_{12} N_2} \right) \\ \frac{dN_2}{dt} &= r_2 N_2 \left( 1 - \frac{N_2}{K_2 + b_{21} N_1} \right). \end{aligned}$$

What type of ecological interactions does the system describe? Draw the null clines and determine the steady states and their stability.