

WEEK 3: EVOLUTIONARY GAME THEORY
Assignments for Mathematical Biology, VT19

1. Consider the waiting game in which two players each pick a waiting time $t_1, t_2 > 0$. Each player pays cost c per unit time. The game ends when $T = \min(t_1, t_2)$ and the payoffs are given by

$$\pi_1(t_1, t_2) = \begin{cases} V - ct_2 & \text{if } t_1 > t_2 \\ V/2 - ct_1 & \text{if } t_1 = t_2 \\ -ct_1 & \text{if } t_1 < t_2 \end{cases}$$

Find an ESS for this game. (Hint: the Bishop-Cannings theorem is applicable to games with continuous strategy space.)

2. Consider the Public Goods Game in which the benefit contributed by cooperators is discounted with a factor w for each additional cooperator, i.e. the payoff for a defector in a group of size N containing k cooperators is given by

$$\pi_D(k) = \frac{rc}{N}(1 + w + w^2 + \dots + w^{k-1})$$

and

$$\pi_C(k) = \pi_D(k) - c.$$

Assuming binomial sampling when the groups are formed write down the expected payoff for cooperators and defectors as a function of x the fraction of cooperators.

Classify the different possible dynamics of the corresponding replicator equation for different values of w . You may fix $N = 10$, $r = 3$ and $c = 3$.