Week 4: Adaptive dynamics

Assignments for Mathematical Biology, VT19

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1. Consider the following model for a population which engages in cannibalism

$$\frac{dx}{dt} = rx - kx^2 + (1-\epsilon)kx^2. \tag{1}$$

The population would, in the absence of cannibalism, grow exponentially with rate r. The negative effect of cannibalism, the additional death rate, is incorporated in the term $-kx^2$. The positive effect, the production of offspring from energy obtained by intra-specific predation, is represented by the term $(1-\epsilon)kx^2$.

Now assume that the parameter k is subject to evolution and using the adaptive dynamics framework describe how k evolves. Is the conclusion reasonable? If not, how could the model be improved.

2. Consider the following model in which two species can alter the carrying capacity of one another:

$$\begin{aligned} \frac{dx_1}{dt} &= r_1 x_1 \left(1 - \tfrac{x_1+x_2}{K_1(x_1,x_2)}\right) - \delta x_1 \\ \frac{dx_2}{dt} &= r_2 x_2 \left(1 - \tfrac{x_1+x_2}{K_2(x_1,x_2)}\right) - \delta x_2, \end{aligned} \tag{2}$$

where

$$K_{1,2}(x_1,x_2) = \gamma k_{1,2} + \frac{1-\gamma}{x_1+x_2}(k_1 x_1 + k_2 x_2), \tag{3}$$

and $r_{1,2} > 0$ and $\delta > 0$. What is the meaning of the parameter γ? Consider species 1 as the resident and 2 as the mutant. Derive an expression for the invasion fitness.

Now consider a trade-off between the growth rate r and the carrying capacity k of the following form $r = f(k) = r_0 e^{-ak}$, where $r_0 > \delta$. Find all singular points of the selection gradient and classify them according to evolutionary and convergence stability.