

WEEK 6: STRUCTURED POPULATION MODELS
Assignments for Mathematical Biology, VT19

1. Let $n(a, t)$ describe the stage distribution in a population which evolves according to

$$\frac{\partial n}{\partial t} + v \frac{\partial n}{\partial a} = -\mu(t)n,$$

where v is a constant rate of maturation and $\mu(t)$ is the death rate. The individuals reproduce according to binary fission at $a = 1$ yielding the boundary condition $n(0, t) = 2n(1, t)$. The initial condition for the system is given by $n(a, 0) = f(a)$. Assuming that $t \gg a$ derive an ODE for how the total population size

$$N(t) = \int_0^1 n(a, t) da$$

evolves in time.

2. Formulate a size-structured model of a population that engages in cannibalism. You may assume that individuals only consume smaller bodied victims and that only a fraction of the required energy to survive and grow comes from cannibalism. Make sure to motivate all other assumptions made.