Berkovich spaces, 7.5 hp

Course period:
January 18 - May 13, 2016

Last day for application:
January 18, 2016

Course leader / Address for applications:
LEADER: Mattias Jonsson / mattiasj@umich.edu (will be a guest professor in Spring 2016)
LOCAL CONTACT: Dennis Eriksson /dener@chalmers.se

Course description (Advertisement for Ph.D. students):
Berkovich spaces are analogues of complex manifolds that appear when replacing complex numbers by the elements of a general normed field, e.g. p-adic numbers or formal Laurent series. They were introduced in the late 1980's by Vladimir Berkovich as a more honestly geometric alternative to the rigid spaces earlier conceived by Tate. In recent years, Berkovich spaces have seen a large and growing number of applications to complex analysis, tropical geometry, complex and arithmetic dynamics, the local Langlands program, Arakelov geometry etc.

We will meet once a week for approximately 14 weeks. There will be homework assignments and an oral exam at the end of the course. The exact schedule will be determined before the course starts.

Responsible department and other participation departments/organisations:
Mathematics Department

Teacher:
Mattias Jonsson, mattiasj@umich.edu

Examiner:
Dennis Eriksson, dener@chalmers.se

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1. Confirmation
The syllabus was confirmed by the Head of the Department of XXX 200X-XX-XX, 200X-XX-XX.

Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences
Main field of study: Mathematics

2. Position in the educational system
Elective course; third-cycle education

3. Entry requirements
While Berkovich spaces are to be viewed as analytic spaces, many of the tools used for studying them tend to be algebraic. For this reason, some familiarity with commutative algebra and algebraic geometry is necessary (the first three chapters of Hartshorne, excluding sheaf cohomology, is more than enough), as is some basic functional analysis. Prior knowledge of other topics (complex manifolds, Banach algebras, p-adic numbers, rigid geometry etc.) may prove useful, but is by no means indispensable.

4. Course content
The first part of the course will be devoted to the basic theory of Berkovich spaces (affinoids, gluing, analytifications). In the second part, we will discuss various applications of specialized topics, partly depending on the interests of the audience.

5. Outcomes

After completion of the course the Ph.D. student is expected to be able to:

- Understand the basic theory of Berkovich spaces and its connections to various other parts of mathematics.

6. Required reading
The following texts are recommended:

- V. G. Berkovich, *Spectral theory and Analytic Geometry over non-Archimedean Fields*. This book, known as the "red book", is quite minimalistic. The teacher will supply some commentaries.
- V. G. Berkovich, *Étale cohomology for some non-Archimedean analytic spaces*. Publ. Math. Inst. Hautes Études Sci. **78** (1993), 5–161. We will likely not do any étale cohomology but, in this paper, Berkovich re-does some of the foundational material.
- S. Bosch, U. Guentzer and R. Remmert, *Non-Archimedean Analysis*. Featuring Atacama-rivaling dryness, this book is not really meant as a textbook, but serves as a good reference for some of the underlying analysis and algebra.

7. Assessment
There will be a few homework sets, containing problems of various levels of difficulty. In addition, there will be an oral exam.

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale
The grading scale comprises Fail, (U), Pass (G)

9. Course Evaluation
The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

10. Language of instruction
The language of instruction is English.