Introduction to Inverse and ill-posed problems, 7.5 Hp

(short course description)

Course period: 01.11.2019-16.02.2019, 08.06-20.06.2020

Course organizers and teachers:
L. Beilina (larisa@chalmers.se), M. Cristofol (michel.cristofol@univ-amu.fr)

Examiner: L. Beilina

Course description

Inverse and ill-posed problems arise in many real-world applications including medical microwave, optical and ultrasound imaging, MRT, MRI, oil prospecting and shape reconstruction, nondestructive testing of materials and detection of explosives, seeing through the walls and constructing of new materials, reconstruction of parameters in biological and chemical models.

Usually, inverse problem consists in recovering of coefficients in some Partial Differential Equation (PDE) or Ordinary differential Equation (ODE), determining of initial conditions or the source function in PDE by knowing the solution of this PDE inside the computational domain or on a part of its boundary. Typical ill-posed problem is the solution of Fredholm integral equations of the first kind.

In this course will be considered in a first step some different theoretical approaches for the inverse reconstruction of unknown coefficients in a PDE by studying a toy problem.

In a second step, physical formulations leading to ill- and well-posed problems, methods of regularization of inverse problems and numerical methods of solution of inverse and ill-posed problems, such that Lagrangian approach and adaptive optimization, methods of analytical reconstruction and layer-stripping algorithms, least-squares algorithms and classification algorithms will be addressed. Numerical solution of ill-posed problems including methods of image reconstruction with applications in image deblurring and magnetic resonance imaging (MRI) will be presented. Machine learning classification algorithms for solution of inverse and ill-posed problems will be also studied as additional tool for improving of solutions obtained in all above described numerical methods.

Introduction to Inverse and ill-posed problems, 7.5 Hp

Confirmation
The course syllabus was confirmed by the Head of the Department of Mathematical Sciences on 20XX-XX-XX.

Field of education: Science 100 %.
Responsible department: Department of Mathematical Sciences.
Main filed of study: Applied Mathematics

Position in the educational system
Elective course; third-cycle education

Entry requirements: Numerical analysis, partial differential equations, finite element method, finite difference method, programming in Matlab

Course description

Inverse and ill-posed problems arise in many real-world applications including medical microwave, optical and ultrasound imaging, MRT, MRI, oil prospecting and shape reconstruction, nondestructive testing of materials and detection of explosives, seeing through the walls and constructing of new materials.

In a first step, we will begin to give a short survey on the domain of Inverse Problems. After a general introduction we will focus on parabolic operators and we will present around the same toy problem different technics:
- the Dirichlet to Neumann methodology,
- the Carleman inequalities approach.

In the second part of the course will be considered physical formulations leading to ill- and well-posed problems, methods of regularization of inverse problems and numerical methods of solution of inverse and ill-posed problems, such that Lagrangian approach and adaptive optimization, methods of analytical reconstruction and layer-stripping algorithms, least-squares algorithms and classification algorithms. Numerical solution of ill-posed problems including methods of image reconstruction with applications in image deblurring and magnetic resonance imaging (MRI) will be presented. Machine learning classification algorithms for solution of inverse and ill-posed problems will be also studied as additional tool for improving of solutions obtained in all above described numerical methods.

This course includes the course project consisting of several assignments where some inverse or ill-posed problem should be solved in Matlab or in C++/PETSc by algorithms studied in the course. During project implementation students will have possibility study how to use toolkit for scientific computations PETSc in C++. When doing project assignments the students get experience in implementation and evaluation of different algorithms for solution of inverse and ill-posed problems.

Learning outcomes

After a successful completion of the course the students will be able to:

Knowledge and understanding

- basic understanding of the notion of inverse problems and the constraints associated
- handling some classic tools in the theory of inverse problems
- understand basic numerical methods for solution of inverse and ill-posed problems.
- derive and use the numerical techniques needed for a professional solution of a given inverse or ill-posed problem.

Skills and abilities

- use computer algorithms, programs and software packages to compute solutions to current inverse or ill-posed problem.
- critically analyze and give advice regarding different choices of regularization techniques, algorithms, and mathematical methods for solution of inverse or ill-posed problem with respect to efficiency and reliability.
- critically analyze the accuracy of the obtained numerical result and present it in a visualized way.
- write a scientific report and make a scientific presentation summarizing obtained results.

Assessment

The examination consists of the computer project and an final oral exam at the end of the course. A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department. In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

Grading scale

The grading scale comprises Fail, (U), Pass (G)

Course Evaluation

The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

Language of instruction

The language of instruction is English.