Introductory K-theory, 7.5 hp

Course period:
February 6 - April 28, 2017

Last day for application:
February 6, 2016

Course leader / Address for applications:
Magnus Goffeng / goffeng@chalmers.se

Course description (Advertisement for Ph.D. students):

K-theory was originally introduced by Grothendieck in the 50's for formulating and proving his groundbreaking version of the Riemann-Roch theorem. It has since been studied, generalized and applied thoroughly. For instance by Atiyah who placed K-theory in an algebraic topological form for usage in the proof of Atiyah-Singer's celebrated index theorem. Later Quillen laid the groundwork for higher algebraic K-theory. Both Atiyah and Quillen received the Fields medal for their efforts.

Roughly speaking, the K-theory group of a topological space is the abelian group generated by all vector bundles. The starting point is the Serre-Swann theorem which relates vector bundles on a space with projective modules over the ring of continuous functions.

This course is on K-theory, both topological and algebraic. Starting with the elementary algebraic notions, the course will treat the fundamental properties of lower dimensional algebraic K-theory and K-theory of (local) Banach algebras. The focus will be on the basic techniques and examples. The main examples will be integers in number fields, group algebras and some low-dimensional manifolds.

The course will start in February and run twice a week (2 hours) until the end of April (LP3-4) for a total of 8 weeks. The schedule will be decided by participants at an introductory meeting.

Responsible department and other participation departments/organisations:
Mathematics Department

Teachers:
Dennis Eriksson and Magnus Goffeng

Examiner:
Magnus Goffeng

Introductory K-theory 1, 7.5 hp

1. Confirmation
The syllabus was confirmed by the Head of the Department of XXX 200X-XX-XX, 200X-XX-XX.

Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences
Main field of study: Mathematics

2. Position in the educational system
Elective course; third-cycle education

3. Entry requirements
Functional analysis, commutative algebra, multilinear algebra, topology

4. Course content
The course will cover a suitable subset of the following topics. The final curriculum will be decided upon during the course.

- Gelfand duality and Serre-Swann's theorem
- Projective modules and algebraic K_0
- K-theory for local Banach algebras
- Higher topological K-theory, periodicity and exactness
- Index theory and the Atiyah-Jänich theorem
- Low-dimensional algebraic K-theory
- Algebraic K-theory of integers of number fields

5. Outcomes
At the end of the course, the students can apply the main results and elementary techniques of K-theory, both topological and algebraic, to simpler examples.

6. Required reading
There are many good books on K-theory, focussing on different aspects of the topic. The following list is a selection with no claims to completeness.

- *K-theory*, Michael Atiyah
- *K-theory for operator algebras*, Bruce Blackadar
- *Elements of noncommutative geometry*, José M. Gracia-Bondia, Joseph C. Varilly, and Héctor Figueroa
- *K-theory, An introduction*, Max Karoubi
- *Algebraic K-theory and its applications*, Jonathan Rosenberg
- *An introduction to K-theory for C^*-algebras*, Mikael Rørdam, Flemming Larsen, and Niels Jakob Laustsen.
- *K-theory and C^*-algebras*, Niels Erik Wegge-Olsen

7. Assessment
There will be bi-weekly homework sheets, and an oral exam at the end of the course.

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale
The grading scale comprises Fail, (U), Pass (G)

9. Course Evaluation
The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

10. Language of instruction
The language of instruction is English.