ON THE FOKKER-PLANCK OPERATOR AS AN ASYMPTOTIC LIMIT AND FINITE ELEMENT METHODS FOR THE VLASOV-POISSON-FOKKER-PLANCK SYSTEM

MOHAMMAD ASADZADEH[1]

Abstract. This talk is about the Fokker-Planck operator (FPO) and some aspects of the finite element methods for the Coulomb plasmas involving the FPO, namely, Vlasov-Poisson-Fokker-Planck System (VPFP). One important application of the FPO is in radiation cancer therapy. VPFP is used in modeling a variety of applied problems ranging from design of computer chips to tokamak and space chattels. Here is a motivation for the need of the study of the FPO in radiation therapy and description of its derivation.

The use of X-rays in radiation therapy began a few days after their discovery. Wilhelm Röntgen announced the discovery of X-rays on December 28, 1895, and Emil Grubbe used them for cancer therapy on January 12, 1896. X-rays are still the most common form of radiation used for cancer therapy, but beams of electrons, protons and other particles are used as well. Current treatment planning in radiation oncology involves computational algorithms based on a mathematical model of the radiation particle beams known as the Fermi equation. Fermi equation is an asymptotic limit of the Fokker-Planck (FP) equation which is yet another asymptotic limit of the linear kinetic, or transport, equation. Therefore, FP is a more accurate model than Fermi. Below is a reasoning for why the original equation in not used.

The description of particle scattering interactions with the background medium is a linear integral operator in the transport equation, and the kernel of this operator describes the probability of scattering from an initial (before scattering) particle energy and direction to a final (after scattering) energy and direction. In certain applications, in particular in radiation therapy using charged particles, this kernel is very peaked about both zero energy transfer and zero direction change and the number of scattering collisions are very large. That is, the average distance a particle travels between the scattering events (the scattering mean free path) is very small. To solve such a scattering transport problem numerically using deterministic methods is very difficult since the mesh size in such a calculation must be on the same scale as the mean free path. This implies an unrealistically fine degree of resolution. Likewise a Monte-Carlo simulation is very time consuming since a very large number of scattering interactions must be followed for each particle before it demise by either absorption or leakage out of the system. To circumvent these difficulties, it has been suggested to replace the integral scattering operator in the transport equation with a differential Fokker-Planck operator. The effect of this replacement is that the dominant (large) in and out scattering terms cancel, thus effectively increasing the mean free path. The classical contributions in this regard are summarized by **Chandrasekhar**, and formalized by **Pomraning**. Part of my research interest concerns some analytical/numerical aspects in this approach.

[1] This research was supported by the Swedish Foundation of Strategic Research (SSF), Gothenburg Mathematical Modeling Centre (GMMC) and Swedish Research Council (VR).