

## Delkurs 2 : extra problems

1. The volume of a closed cylinder, measured in cubic metres, equals twice its surface area, measured in square metres.

(i) Compute the general relationship which this implies between the height  $h$  and the radius  $r$  of the cylinder

(ii) Deduce that any such cylinder must have radius greater than 4 metres

(iii) If the radius is 6 metres, compute the volume of the cylinder (in terms of  $\pi$ ).

2. You have three pyramids A,B and C, each with a square base. B is similar to (likformig med) A and  $2/3$  as high. C is only  $1/2$  as high as A, but has  $4/5$  of its' base length. Which of B and C has the greater volume ? What is the exact ratio of the volumes of B and C ?

3. In a square of side length 100, circles of radius 1 are packed as tightly as possible without overlapping.

(i) what fraction of the area of the square is covered by the circles ?

\* (ii) suppose now you're given one further circle, also of radius 1, and allowed to place it wherever you like. How should you position it in order to cover as much as possible of the remaining area ? How much extra area will you cover in that case ?

(OBS! The answer to the first question in (ii) is, I think, "intuitively obvious", but perhaps quite tricky to prove rigorously, so I don't expect you to do the latter. However, there still remains a computation to answer the second question).

4. A regular octagon is inscribed in a circle of radius 1. Compute its area and circumference. Use the latter to deduce a lower bound for  $\pi$ .

(TIPS : Use trigonometry and/or Pythagoras).

5. The side lengths of a rectangular box are 4,6 and 7 metres. What is the distance between a pair of opposite corners ?

6. In another box, similar to that in **Q.5**, the length of the shortest side is 7 metres. What are the lengths of its other sides ? What is the ratio of the surface area (resp. volume) of the new box to that of the old one ?

7. A cylinder of diameter 10 metres and height 8 metres is inscribed in a cone of height 20 metres. What is the volume of the cone ?

8. Jane is 1,84 metres tall. When she stands 40 metres behind a 5m tall telegraph pole, positioned at a street corner, and looks directly over the top of the pole, then she can just see the top of a very tall building 10 blocks away. If the average length of a city block is about 200 metres

(i) how much further back (approximately) would Jane have to go if she wanted to lie on the ground and still be able to just see the top of the building while looking directly over the telegraph pole ?

(ii) how tall (approximately) is the building ?