SUGGESTION FOR BACHELOR THESIS PROJECT

The field of the proposed project is *opinion dynamics* which involves the mathematical modeling of how groups of agents (people) influence one anothers' opinions. This is very much a cross-disciplinary field, involving work by physicists, computer scientists, robotics engineers and, of course, social scientists. Mathematically, it is still very young, with a shortage of rigorous theorems even at a very basic level. However, interest from mathematicians is slowly growing, and there are several people at this department working on various aspects of the field.

More precisely, the proposed project is concerned with studying a class of models introduced by Rainer Hegselmann and Ulrich Krause 12-15 years ago. In its most well-known form, the Hegselmann-Krause model [1] posits that there are a finite number of agents, indexed by the integers $1, 2, \ldots, n$. Time is measured discretely, and the opinion of agent i at time t is represented by a real number $x_t(i)$. At each time step, all agents simoultaneously update their opinions according to the formula

$$x_{t+1}(i) = \frac{1}{|\mathcal{N}_t(i)|} \sum_{j \in \mathcal{N}_t(i)} x_t(j),$$

where $\mathcal{N}_t(i) = \{j : |x_t(j) - x_t(i)| \leq 1\}$. In words, each agent updates his opinion to the average of those which currently lie within unit distance of his own.

The simplicity of this model makes it very appealing[1]. Nevertheless, it has been surprisingly difficult to prove rigorous results about the dynamics. My graduate students and I have several recent results [2]-[5], which are very much state-of-the-art for this field.

Specifically for this Bachelor project, in addition to learning about the model, and variations on it, in general (which in a long-term perspective is the most important part), I would suggest a concrete problem which arises in our latest preprint [5]. There we have a result which we know holds for all sufficiently large positive integers n - see Theorem 3.1. We would like to know for exactly what values of n the result holds, and suspect this can be decided with enough hard work.

For more detailed information, contact Peter Hegarty, hegarty@chalmers.se

[1] R. Hegselmann and U. Krause, *Opinion dynamics and bounded confidence: models, analysis and simulations*, Journal of Artificial Societies and Social Simulation **5** (2002), No. 3. Fulltext at `http://jasss.soc.surrey.ac.uk/5/3/2/2.pdf`

Papers 2-5 are all preprints. For the latest versions of all these papers, consult my homepage: http://www.math.chalmers.se/∼hegarty/research.html. Reading the introductions to these papers will also give you a very good idea of the state of the field.

[2] E. Wedin and P. Hegarty, *The Hegselmann-Krause dynamics for continuous agents and a regular opinion function do not always lead to consensus.*

[3] P. Hegarty and E. Wedin, *The Hegselmann-Krause dynamics for equally spaced agents.*

[4] E. Wedin and P. Hegarty, *A quadratic lower bound for the convergence rate in the one-dimensional Hegselmann-Krause dynamics.*

[1]This simplicity is due in part to the fact that the update rule is completely deterministic, plus there are no "geographical" limits on sharing of opinions. There are other classes of models in which both random and geographical features appear, and these are also the subject of active study by other groups at this department.

[5] P. Hegarty, A. Martinsson and E. Wedin, *The Hegselmann-Krause dynamics on the circle converge.*