SUGGESTION FOR BACHELOR THESIS PROJECT: TERMINATOR 2, MULTIAGENT RENDEZVOUS AND THE DYNAMICS OF OPINION FORMATION

This is a theoretical project, which means you probably won't actually build anything, though you might write some software to implement an algorithm. In presenting the project for a Z-student, it is perhaps best to start from the so-called *mulitagent rendezvous problem*, which is a problem in robotics, more precisely in distributed control of autonomous agents.

Suppose a collection of identical robots are scattered in the plane $\mathbb{R}^2$ (imagined as representing a piece of the earth's surface, say). Each robot is equipped with a sensor which allows it to determine the precise location of other robots within a certain fixed radius of itself, where the radius depends on the sensitivity of the equipment, but is the same for all the robots. The task the robots have to solve is to all meet up (rendezvous) at the same location - it doesn't matter where exactly, just that they all congregate somewhere. What makes the task difficult is that (i) each robot has only a finite viewing range, hence, if there are many robots scattered over a wide area, no individual has global knowledge of the configuration (ii) the robots are identical, so there is no designated "leader" who can order the others what to do. These are the classic conditions for a *distributed control* problem.

It turns out that there are efficient algorithms for solving the rendezvous problem, assuming of course that initially no subset of the robots is completely isolated from its complement. However, even the simplest algorithms are quite sutble in the details, and not exactly the first thing one might think of. The most famous procedure is known as the ASY-algorithm[1], and dates from 1999, but I will not describe it here. Instead, let me mention what *might* be the first thing one would think of[2], namely:

Rule 1: "At each step, each robot should observe the positions of those inside its viewing range and move to the average of their positions. Then iterate this procedure ...".

It turns out, perhaps surprisingly, that this procedure often fails, different clusters of robots can become isolated from one another. However, suppose we now change perspective and, instead of dealing with programmable robots, we are dealing with intelligent human agents obeying nothing other than their own free will. Rule 1 is a famous mathematical model in the field of *opinion dynamics*, which is concerned with how groups of agents (people) influence one anothers' opinions. Here, points in the plane are an abstract representation of the opinions - in fact, in this setting it is simpler to work in just one dimension, so opinions are just real numbers. The "finite viewing range" is now reinterpreted as a "finite confidence range", meaning that each person is only willing to compromise with those whose opinions lie within a certain distance of their own. To social scientists, Rule 1 is known as the *Hegselmann-Krause bounded confidence model*[3]. A mathematically rigorous formulation of the one-dimensional model is as follows: There are a finite number of agents, indexed by the integers $1, 2, \ldots, n$. Time is measured discretely, and the opinion of agent i at time t is represented by a real number $x_t(i)$. At each time step, all

[1] Ando-Suzuki-Yamashita.

[2] It *was* the first thing I thought of, when I first encountered this problem, having been inspired by a scene from Terminator 2, no less !

[3] It also appeared in the literature for the first time around 15 years ago, but the authors seem to have been completely unaware of the rendezvous interpretation. This connection has since been well established, primarily by control engineers researching in this area.

agents simoultaneously update their opinions according to the formula

$$x_{t+1}(i) = \frac{1}{|\mathcal{N}_t(i)|} \sum_{j \in \mathcal{N}_t(i)} x_t(j),$$

where $\mathcal{N}_t(i) = \{j : |x_t(j) - x_t(i)| \leq 1\}$. In words, each agent updates his opinion to the average of those which currently lie within unit distance of his own.

Edvin Wedin and myself have been working on this model for the last year or so, and have proven a number of theorems about it, see the references below. Indeed, mathematically this apparently simple model is still poorly understood and proofs are still being sought for many fundamental phenomena which have been observed in simulations. The introductions to our papers are hopefully legible and give a good overview of the current state of knowledge[4].

Z-students are welcome to work on the opinion dynamics aspects of this project (which is an ongoing research project in the math department), but may be more interested in the control engineering aspects which we started out from. In addition to learning about the field in general, and studying the existing literature, some specific possible goals for a project with the latter focus are given below. Note, however, that this is an active research field, and we are quite open to the idea of students following their own alternative inclinations.

Idea 1: Learn the ASY-algorithm and implement it graphically.

Idea 2: Simulate the 2-dimensional Hegselmann-Krause model. So far, Edvin and I have only coded for the one-dimensional model. In higher dimensions, the dynamics are more subtle, for example groups of agents which are isolated from one another can get back into contact in ways which cannot happen in 1-D. Hence, it is expected that some genuinely new phenomena will appear. It would make a very valuable contribution to our current research to have good simulation code in two and even higher dimensions.

Idea 3: Recently, a group of German researchers have published papers dealing with “opinion control” in the HK-model. The idea here is that someone, say a marketer or a political lobbyist, tries to “plant” opinions in such a way as to influence the dynamics in a way they desire. This is a new way of looking at opinion dynamics as a control engineering problem, and may lead to exciting new discoveries. A project could involve studying these new papers, summarising them and maybe running some simulations of your own (or even proving a new theorem in the best case !).

Idea 4: The most ambitious, but also the least well-defined, idea we have is to move beyond the rendezvous problem per se, and look at other kinds of distributed control problems. In particular, there was a big media fuss recently about this topic:

http://www.bbc.com/news/science-environment-28739371

The project could involve studying their original paper, [5] below, as well as earlier references, and describing their algorithm, perhaps even implementing it (on a computer) for a smallish number of agents. Obs! Most of the literature concerning Idea 4 is not very mathematical in nature. This may or may not suit you, but it is also likely that Edvin and I are less familiar with it and you may find us learning along with you !

For more detailed information, contact Peter Hegarty, hegarty@chalmers.se

Prerequisites: Apart from linear algebra, thee aren't really any definitive mathematical prerequisites, there isn't any high-powered theory behind this stuff and it is possible to pick up

[4]This field is very young and I don't know of any good textbook, certainly not a good mathematical text.

what you need as you go along. You should be comfortable, however, with reading mathematical texts in general, and have good grades in your math courses.

Papers 1-4 are all preprints. For the latest versions of all these papers, consult my homepage: http://www.math.chalmers.se/∼hegarty/research.html. Reading the introductions to these papers will also give you a very good idea of the state of the field.

[1] E. Wedin and P. Hegarty, *The Hegselmann-Krause dynamics for continuous agents and a regular opinion function do not always lead to consensus.*

[2] P. Hegarty and E. Wedin, *The Hegselmann-Krause dynamics for equally spaced agents.*

[3] E. Wedin and P. Hegarty, *A quadratic lower bound for the convergence rate in the one-dimensional Hegselmann-Krause dynamics.*

[4] P. Hegarty, A. Martinsson and E. Wedin, *The Hegselmann-Krause dynamics on the circle converge.*

[5] M. Rubinstein and W.-M. Shen, *A scalable and distributed approach for self-assembly and self-healing of a differentiated shape*, IEEE/RSJ International Conference on Intelligent Robots and Systems, Nice, France, Sept. 22-26, 2008.