The cross number conjecture – a combinatorial problem in finite abelian groups
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Let $G$ be a finite abelian (additive) group. Call a sequence $S$ of elements $g$ in $G$ minimal if it sums up to $0$ but each proper sub-sum differs from $0$ (repetitions in $S$ allowed). The cross number $k(S)$ of $S$ is defined as the sum of the reciprocal values of the orders of $g$ in $S$. The cross number $k(G)$ of group $G$ is the maximum of all $k(S)$, $S$ a minimal sequence.

Let the decomposition of $G$ into cyclic groups of prime power consist of $C(i)$ with order $n(i)$ for $i=1,...,s$. Then the cross number conjecture states that $k(G) = L - R + s$, where $L$ is the reciprocal of the lowest common multiple of the $n(i)$ and $R$ is the sum over the reciprocals of $n(i)$.

Up to now this conjecture has been neither proven nor disproven. The talk will present cases of $G$ for which the conjecture is true, an example being cyclic groups of prime power order. But even for finite cyclic groups in general the conjecture is still open.

The talk will also address origin and importance of the conjecture with respect to the ideal class group of an algebraic number field and its arithmetical implications.