Proposition *Let* $n \geq 2$. *Then*

$$d_n = (n-1)(d_{n-1} + d_{n-2}). \tag{1}$$

PROOF : Let π be a derangement of the numbers $1, 2, ..., n$. There are $n-1$ choices for $i = \pi(1)$. Fix i. Then there are two possibilities :

(A) $\pi(i) = 1$. Then π induces a derangement of the numbers $2, 3, ..., i-1, i+1, ..., n$, so there are d_{n-2} possibilities left for π.

(B) $\pi(i) \neq 1$. Then, identifying 1 and i, we can imagine that π induces a derangement of $2, 3, ..., n$, so there are d_{n-1} possibilities left for π.

Thus, once i is fixed, there are in total $d_{n-1} + d_{n-2}$ possible derangements. And since there were $n-1$ choices for i, an application of the multiplication principle yields (1).