Exercise For each $n \geq 0$, let A_n be the number of triangulations of an $(n+2)$-gon. Prove that $A_n = C_n$ for all n.

Solution It suffices to prove that the A_n satisfy the same recurrence relation as the C_n, namely that

$$A_0 = 1,$$
$$A_n = \sum_{m=1}^{n} A_{m-1}A_{n-m} \quad \text{for all } n \geq 1.$$

It's clear that $A_0 = 1$. To prove the recurrence relation, label the vertices of an $(n+2)$-gon as $1, 2, ..., n+2$ in some order. I claim that $A_{m-1}A_{n-m}$ is the number of triangulations in which the edge $\{n+1, n+2\}$ is joined to the vertex m. For the triangle $\{n+1, n+2, m\}$ divides the remainder of the $(n+2)$-gon into two halves, consisting of $m+1$ and $n-m+2$ vertices respectively. So, by definition, there are A_{m-1} and A_{n-m} possible triangulations of the two halves respectively. Finally, apply MP.