

## List of examinable proofs

The following is a list of the theorems from the lecture notes which you may be asked to prove on the exam. Approximately 60 percent of the marks on the exam will be for proofs of theorems on this list.

Note that the proofs of many theorems build upon one another. Whenever this is the case, I indicate whether auxiliary results need to be proven or just quoted in order to get full points.

Theorem 3.5 (you may quote FTA).

Theorem 3.8.

Theorem 4.1 (you may quote Theorem 3.8).

Corollary 5.5 (you may quote Theorem 5.3).

Theorem 6.1 (only lower bound for  $\pi(x)$  is examinable).

Theorem 6.4 (you may quote Prop. 6.2).

Theorem 7.2 and Corollary 7.3.

Proposition 7.8.

Supplementary Week 47: Theorem 1.7.

Proposition 8.1.

Theorems 9.3 and 9.6 (you must prove Props. 9.1, 9.2, Lemma 9.5 if used).

Theorem 10.2 (you may quote Lemma 15.3).

Theorem 11.7 (you may quote Prop. 12.2 and Theorem 12.4).

Theorem 12.4.

Corollary 12.6 (you may quote Theorem 12.4).

Theorem 15.2 (you may quote Prop. 14.1 and Lemmas 14.2, 14.3, 15.1).

Supplementary Week 49: Theorem 1.1(i) (you may quote the Abel summation formula)

Supplementary Week 49: Proof that  $L(1, \chi) \neq 0$  for non-trivial real  $\chi$  (you

2

may quote Theorem 1.1 and Lemma 1.2).

Theorem 17.6 ( $h = 2$  only).

The theorem that in any finite  $S \subseteq \mathbb{Z} \setminus \{0\}$ , there exists a sum-free subset of size at least  $\frac{|S|+1}{3}$ .

The upper bound  $f(n) \leq \log_2 n + \frac{1}{2} \log_2 \log_2 n + O(1)$  on the size of a subset of  $\{1, \dots, n\}$  with distinct subset sums. You may assume Chebyshev's inequality.

Theorem 18.2 ( $h = 2$  only. Chernoff's inequality plus the Borel-Cantelli lemma may be assumed without proof, as may the calculus exercise to estimate  $\mu_n$ ).

Szemerédi Regularity Lemma: you will NOT be asked to prove this. You should know the statement though.

Roth's theorem: you should know all steps of the proof, starting from the Regularity Lemma, which you may assume.