Modular Forms and Generating Series

7.5 Course Credits (ECTS)

GENERATING series are one useful way to organize information in mathematics, which has proved useful in plenty of fields. For example, combinatorics, representation theory, discrete, enumerative and arithmetic geometry, and many others. Generating series are particularly useful, if they have special analytic properties. The easiest case are polynomials, then rational functions. Modular forms can be viewed as the next step of complications.

In this course, we will: (1) Introduce generating series, exemplified by a generating function for integer partitions from combinatorics; (2) Introduce classical modular forms and computationally experiment with some modular generating series; (3) Introduce vector valued modular forms and thus obtain closed formulas for some combinatorial quantities.

Aim of the Course

Students, after following this course, should be able to produce generating series from basic combinatorial questions, check computationally whether they might be modular forms, and deduce relations for them in case they are.

Duration

The course will take place in LP2 and LP3 of 2015/16. We will meet once per week for theoretical work, and later additionally once per week for computational work.

Prerequisites

Basic knowledge of complex analysis is necessary. Students are expected to work with computers. Prior knowledge of Sage or Python is helpful, but not required. Basic understanding of combinatorics is helpful, but not required, either.

Lecturers and Course Organizers

Martin Westerholt-Raum, raum@chalmers.se, office L2130, phone 3522.

Lectures and Examination

The Course will start early November. An announcement for a pre-meeting will be sent out to students who showed interest. We will try to find dates that fit everybody.

During the course there will be homework assignments. Further, students will, altogether, implement a small program in the computer algebra system Sage. At the end of the course there will be an oral exam.

Literature

Lecture notes will be distributed during the course. The following literature complements them:

- (1) generatingfunctionology, Second Edition, *Chapter 1&2*, Herbert Wilf. Download at https://www.math.upenn.edu/~wilf/DownldGF.html.
- (2) Some applications of modular forms, *Chapter 1*, Peter Sarnak. Cambridge Tracts in Mathematics, 99. Cambridge University Press, Cambridge, 1990. ISBN: 0-521-40245-6.

(3) The 1-2-3 of modular forms, *Chapter 1*, Don Zagier. Lectures from the Summer School on Modular Forms and their Applications held in Nordfjordeid, June 2004. Edited by Kristian Ranestad. Universitext. Springer-Verlag, Berlin, 2008. ISBN: 978-3-540-74117-6.

Registration

Please send email to the course organizer for registration.