Modular Forms and Generating Series, 7.5 hp

Course period:
November 2, 2015 - March 11, 2016

Last day for application:
November 2, 2015

Course leader / Address for applications:
Martin Westerholt-Raum / raum@chalmers.se

Course description (Advertisement for Ph.D. students):
Generating series are one useful way to organize information in mathematics, which has proved useful in plenty of fields. For example, combinatorics, representation theory, discrete, enumerative and arithmetic geometry, and many others. Generating series are particularly useful if they have special analytic properties. The easiest case are polynomials, then rational functions. Modular forms can be viewed as the next level of complication.

The course will take place in LP2 and LP3 of 2015/16. We will meet once a week for theoretical work, and later additionally once per week for computational work. The exact schedule will be determined before the course starts.

Responsible department and other participation departments/organisations:
Mathematics Department

Teachers:
Martin Westerholt-Raum, raum@chalmers.se

Examiner:
Martin Westerholt-Raum

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1. Confirmation
The syllabus was confirmed by the Head of the Department of XXX 200X-XX-XX, 200X-XX-XX.

Disciplinary domain: Science
Department in charge: Department of Mathematical Sciences
Main field of study: Mathematics

2. Position in the educational system
Elective course; third-cycle education

3. Entry requirements
Basic knowledge of complex analysis is necessary. Students are expected to work with computers. Prior knowledge of Sage or Python is helpful, but not required. Basic understanding of combinatorics is helpful, but not required either.

4. Course content
In this course, we will: (1) Introduce generating series, exemplified by a generating function for integer partitions from combinatorics; (2) Introduce classical modular forms and computationally experiment with some modular generating series; (3) Introduce vector valued modular forms and thus obtain closed formulas for some combinatorial quantities.

5. Outcomes

After completion of the course the Ph.D. student is expected to be able to:

- Produce generating series from basic combinatorial questions, check computationally whether they might be modular forms and deduce relations for them in case they are.

6. Required reading
Lecture notes will be distributed during the course. The following literature complements them:

- generatingfunctionology, Second Edition, *Chapters 1-2*, Herbert Wilf. Download at `https://www.math.upenn.edu/~wilf/DownldGF.html`
- Some applications of modular forms, *Chapter 1*, Peter Sarnak. Cambridge Tracts in Mathematics, 99. Cambridge University Press, Cambridge, 1990. ISBN: 0-521-40245-6.

7. Assessment
During the course there will be homework assignments. Further, students will, altogether, implement a small program in the computer algebra system Sage. At the end of the course there will be an oral exam.

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale
The grading scale comprises Fail, (U), Pass (G)

9. Course Evaluation
The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.

10. Language of instruction
The language of instruction is English.