Course description

Ornstein-Uhlenbeck theory in finite dimension

The classical Hermite polynomials on the real line or in n-dimensional Euclidean space are orthogonal with respect to the Gaussian measure $\exp(-|x|^2)dx$, and they are eigenfunctions of the Hermite operator $H = -\Delta/2 + x \cdot \text{grad}$. This operator is self-adjoint and positive semidefinite. Thus one can define the Ornstein-Uhlenbeck semigroup as $(\exp(-tH))_{t>0}$, spectrally. It is remarkable that an explicit expression for the kernel of this semigroup was found already in 1866 by Mehler. In this setting, one can develop a version of harmonic analysis involving notions like singular integral operators, for instance Riesz transforms such as $\partial/\partial x\, H^{-1/2}$. The kernels of these operators can be expressed by means of the Mehler kernel, and this makes it possible to determine their L^p mapping properties. Maximal functions come in here, in particular that of the semigroup.

One can describe this theory as a model of harmonic analysis where Lebesgue measure is everywhere replaced by the Gaussian measure. Our approach is analytic rather than probabilistic.

The course will introduce this setting and discuss operators of the types mentioned. It will be seen that most of the operators are bounded on the L^p space defined by the Gaussian measure, and some but not all are also of weak type (1,1).