Introduction to Riemannian Geometry, 7.5 hp

Course period: March 20 - May 31, 2017

Course leader:

Genkai Zhang, genkai@chalmers.se

Course description (Advertisement for Ph.D. students):

The primary goal of Riemannian geometry is to study the geometry and topology of manifolds by analytical methods. It is one of the most active fields in the current mathematics research. It is also a common language for many other research areas in mathematics and physics. Ph.D students in physics and control theory are also welcome to take the course.

We shall start with the basics of calculus on manifolds, exterior differentiation, vector fields and distributions on manifolds, Frobenius theorem on integrable distributions. We introduce then some central concepts of Riemannian geometry, Riemannian metrics and connections, geodesics and curvature tensors. Most computable examples for these concepts are provided by Lie groups and symmetric spaces, in particular the spaces of constant curvatures such as projective spaces or hyperbolic spaces. We shall follow partly the books [1-2]. We study then the analytic and topological properties of Riemannian manifolds, such as Hopf-Rinow theorem, the variation of geodesics, Jacobi fields, the sphere theorem, following partly the book [4].

The course will start in the LP4, around the week 12 in March 2017, twice a week till the end of May. The exact schedule will be fixed in the beginning of the course.

Responsible department and other participation departments/organisations: Mathematics Department

Teacher/Examiner Genkai Zhang

Introduction to Riemannian Geometry 1, 7.5 hp

1. Confirmation

The syllabus was confirmed by the Head of the Department of XXX 200X-XX-XX, 200X-XX-XX.

Disciplinary domain: Science Department in charge: Department of Mathematical Sciences Main field of study: Mathematics

2. Position in the educational system

Elective course; third-cycle education

3. Entry requirements

Higher differential calculus, real analysis, topology.

4. Course content

- Differential manifolds and calculus on manifolds.
- Riemannian metric and connections
- Geodesics and Jacobi fields
- Lie groups, symmetric space and spaces of constant curvature

5. Outcomes

Basic knowledge on Riemannian geometry and tools.

6. Required and recommended reading

- 1. Boothby, Introduction to differentiable manifolds and Riemannian Geometry, Academic Press.
- 2. Helgason, Differential Geometry, Lie Groups and Symmetric Spaces. Academic Press.
- 3. Hörmander, Riemannian Geometry, Lund Univ. Lecture Notes.
- 4. Do Carmo, Riemannian Geometry. Birkhäuser.

7. Assessment

There will be a few homework sheets, and an oral exam at the end of the course

A Ph.D. student who has failed a test twice has the right to change examiners, if it is possible. A written application should be sent to the Department.

In cases where a course has been discontinued or major changes have been made a Ph.D. should be guaranteed at least three examination occasions (including the ordinary examination occasion) during a time of at least one year from the last time the course was given.

8. Grading scale

The grading scale comprises Fail, (U), Pass (G)

9. Course Evaluation

The course evaluation is carried out together with the Ph.D. students at the end of the course, and is followed by an individual, anonymous survey. The results and possible changes in the course will be shared with the students who participated in the evaluation and to those who are beginning the course.