Att lämna in måndag den 15 september

1 (10.1.4 in Biggs) Show that in a group of 10 people, there must either be 4 mutual friends or 3 mutual strangers.

2. Without using the formula for the binomial coefficients, prove that, if n is a positive integer then

$$n \cdot \binom{2n}{n} = (n+1) \cdot \binom{2n}{n+1}.$$

(Hint : Show that the two sides count the same thing in two different ways).

3. How many 27-element subsets of the set $\{1, 2, 3, ..., 200\}$ contain no two consecutive integers ?

4 (19.2.5(ii) in Biggs) Without using the formula for the Fibonacci numbers (f_n), prove that

$$f_n f_{n+2} = f_{n+1}^2 + (-1)^n.$$

(Hint : Manipulate the recurrence relation. I'd be very impressed if someone comes up with a 'combinatorial' proof, i.e.: showing that the two sides count the same thing in two different ways).

5. For positive integers n, k let $p(n, k)$ denote the number of partitions of n into k parts. Prove that

$$p(n, k) = p(n-1, k-1) + p(n-k, k).$$

Bonus problem

Use the recurrence relation for the Stirling numbers $S(n, k)$ to find a nice explicit formula for the numbers $S(n, 3)$. Then try to reprove this formula without using the recurrence relation.