TMA 055 : Diskret Matematik (E3)

Week 5

Demonstration problems for Tuesday, Sept 30

1. Compute

$$5^{2003} \pmod{23}$$

using the repeated squaring method.

We have now come to the 'end' of the second part of the course, that dealing with arithmetic/number theory. The following exercises are intended to reinforce some of the ideas introduced over the last couple of weeks, and perhaps fill in a gap or two.

2. The *least common multiple* of the integers a and b, denoted $\mathrm{LCM}(a, b)$, is defined to be the least positive integer m such that both $a|m$ and $b|m$. Using the FTA, explain the following facts :

(a) If n is any common multiple of a and b (i.e.: any integer such that both $a|n$ and $b|n$), then $m|n$.
(b) For any integers a, b we have that

$$\mathrm{LCM}(a, b) = \frac{a \cdot b}{\mathrm{GCD}(a, b)}.$$

3. Prove that $n^3 - n$ is divisible by 6 for all integers n. For which integers is it divisible by 12 ?

4. Find all integer solutions to

$$37x \equiv 3 \pmod{97}.$$

5. Compute $\phi(10585)$. Notice anything ? (I don't really expect you to, but the övningsledare will let you know what I mean !!).

Demonstration problems for Thursday, Oct. 2

1 (15.3.1 in Biggs) Is it possible that the following lists are the degrees of all the vertices of a simple graph ? If so, give a pictorial representation of such a graph.

$$(i)\ 2,2,2,3 \qquad (ii)\ 1,2,2,3,4$$
$$(iii)\ 2,2,4,4,4 \qquad (iv)\ 1,2,3,4$$

2 (see 15.2.1 and 15.8.3 in Biggs) Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs. G_1 and G_2 are said to be *isomorphic* if they have the same number of vertices, i.e.: $|V_1| = |V_2|$, and there is a 1-1 mapping $\alpha : V_1 \to V_2$ which takes edges to edges, i.e.: $\{v, w\}$ is an edge in G_1 if and only if $\{\alpha(v), \alpha(w)\}$ is an edge in G_2.

Show that the first pair of graphs below are not isomorphic whereas the second pair are.

Diagrams missing

Diagrams missing

3. Let $G = (V, E)$ be a simple graph with n vertices and suppose the vertices have been numbered from 1 to n (the graph is said to be *labelled*). Let $M = (m_{ij})$ be the $n \times n$ matrix defined by

$$m_{ij} = \begin{cases} 1, & \text{if } \{i,j\} \text{ is an edge in } G, \\ 0, & \text{otherwise.} \end{cases}$$

M is called the *adjacency matrix* of the labelled graph G.

Write down M for the labelled graph G below.

Diagram missing

Compute M^2, M^3, M^4. Interpret the entries in these matrices in terms of paths in G. Formulate a general result, i.e.: something that applies to all labelled graphs.

4 (15.8.5, 15.8.6 in Biggs) The *k-cube* Q_k is the graph whose vertices are the words of length k in the alphabet $\{0, 1\}$ and whose edges join words which differ in exactly one position. Show that

(i) Q_k is a regular graph of degree k

(ii) Q_k is bipartite

(iii) Q_k has a Hamilton cycle.

Further practice problems

(this list will be constantly updated)

In this part of the course (i.e.: graph theory) I am following Biggs quite closely. Hence I will make photocopies of all the exercises in Chapter 15 of Bigga and hand them out in class. If you don't already have the book, you should therefore get a copy of these exercises from me. I will leave additional copies in the box outside my office door for people to collect.