1. A permutation $a_1, a_2, ..., a_n$ of the integers $1, 2, ..., n$ is said to be $1-3-2$ *avoiding* if there does not exist any three integers i, j, k such that

$$1 \leq i < j < k \leq n$$

and

$$a_i < a_j > a_k > a_i.$$

Write out all 1-3-2 avoiding permutations of $\{1, 2, ..., n\}$ for $n = 1, 2, 3, 4$. Let A_n denote the number of such permutations. Show that $A_n = C_n$, the n:th Catalan number.

(Hint : By considering the position of n in a permutation, show that the A_n satisfy the same recurrence relation as the C_n.)

2. In this exercise, $p(n, k)$ denotes the number of partitions of a positive integer n into k parts and $p(n) = \sum_k p(n, k)$ the total number of partitions of n.

(i) Evaluate $p(8)$.
(ii) Find a formula for $p(n, 2)$.
(iii) Explain why $p(n, n-k) = p(k)$ when $k \leq n/2$.