

1. Let n be a positive integer with prime factorisation

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}.$$

Find a formula for $d(n)$, the number of divisors of n . Hence compute $d(3000)$.

- 2 (see exercises 9.2 (1.8.7) in Biggs)** Use the fundamental theorem of arithmetic (FTA) to prove that \sqrt{p} is an irrational number for any prime p .

- 3.** Find the general solution to the Diophantine equation

$$23x + 41y = 2000,$$

and write down all solutions for which $x > 0$ and $y > 0$.

- 4.** Use FTA to prove that $x = -1, y = 0$ is the only solution to the Diophantine equation

$$y^2 = x^2 + 3x + 2.$$