Övningstenta 2

The homogeneous equation is

$$a_n - 2a_{n-1} - 3a_{n-2} = 0.$$

The characteristic equation for this is

$$x^2 - 2x - 3 = 0,$$

which factorises as

$$(x-3)(x+1) = 0,$$

and hence has the two roots $x = 3$, $x = -1$. Hence the general solution to the homogeneous equation is

$$a_n^h = C_1 \cdot 3^n + C_2 \cdot (-1)^n.$$

Our guess for a particular solution should have the form

$$a_n^p = A \cdot 2^n.$$

Substituting into the recurrence relation, the requirement on A is that

$$A \cdot \left[2^n - 2 \cdot 2^{n-1} - 3 \cdot 2^{n-2} \right] = 2^n,$$

from which we deduce that $A = -4/3$. Hence the general solution to our recurrence relation is

$$a_n = C_1 \cdot 3^n + C_2 \cdot (-1)^n - \frac{4}{3} \cdot 2^n.$$

It remains to insert the initial conditions :

$$n = 0 \Rightarrow a_0 = 1 = C_1 + C_2 - \frac{4}{3},$$
$$n = 1 \Rightarrow a_1 = 1 = 3C_1 - C_2 - \frac{8}{3}.$$

Solving, we obtain $C_1 = 3/2$, $C_2 = 5/6$. Hence the final answer is

$$a_n = \frac{3}{2} \cdot 3^n + \frac{5}{6} \cdot (-1)^n - \frac{4}{3} \cdot 2^n.$$

Övningstenta 3

The homogeneous equation is

$$a_n - 3a_{n-1} = 0.$$

The characteristic equation for this is

$$x - 3 = 0,$$

which hence has the root $x = 3$. Hence the general solution to the homogeneous equation is

$$a_n^h = C \cdot 3^n.$$

Our guess for a particular solution should have the form

$$a_n^p = An + B.$$

Substituting into the recurrence relation, the requirement on A and B is that

$$An + B - 3[A(n-1) + B] = n,$$

from which we deduce that $A = -1/2$, $B = -3/4$. Hence the general solution to our recurrence relation is

$$a_n = C \cdot 3^n - \frac{n}{2} - \frac{3}{4}.$$

It remains to insert the initial condition :

$$n = 0 \Rightarrow a_0 = 1 = C - \frac{3}{4},$$

from which we obtain $C = 7/4$. Hence the final answer is

$$a_n = \frac{7}{4} \cdot 3^n - \frac{n}{2} - \frac{3}{4}.$$

Tenta 20/10/03

The homogeneous equation is

$$a_n - 4a_{n-1} = 0.$$

The characteristic equation for this is

$$x - 4 = 0,$$

which hence has the root $x = 4$. Hence the general solution to the homogeneous equation is

$$a_n^h = C \cdot 4^n.$$

Our guess for a particular solution should have the form

$$a_n^p = An + B.$$

Substituting into the recurrence relation, the requirement on A and B is that

$$An + B - 4[A(n-1) + B] = 2n + 1,$$

from which we deduce that $A = -2/3$, $B = -11/9$. Hence the general solution to our recurrence relation is

$$a_n = C \cdot 4^n - \frac{2n}{3} - \frac{11}{9}.$$

It remains to insert the initial condition :

$$n = 0 \Rightarrow a_0 = 2 = C - \frac{11}{9},$$

from which we obtain $C = 29/9$. Hence the final answer is

$$a_n = \frac{29}{9} \cdot 4^n - \frac{2n}{3} - \frac{11}{9}.$$

Tenta 10/01/04

The homogeneous equation is

$$a_n - 5a_{n-1} + 4a_{n-2} = 0.$$

The characteristic equation for this is

$$x^2 - 5x + 4 = 0,$$

which factorises as

$$(x-1)(x-4) = 0,$$

and hence has the two roots $x = 1$, $x = 4$. Hence the general solution to the homogeneous equation is

$$a_n^h = C_1 + C_2 \cdot 4^n.$$

Since 4^n is already a solution to the homogeneous equation, our guess for a particular solution should have the form

$$a_n^p = A \cdot n \cdot 4^n.$$

Substituting into the recurrence relation, the requirement on A is that

$$A \cdot \left[n4^n - 5(n-1)4^{n-1} + 4(n-2)4^{n-2} \right] = 4^n,$$

from which we deduce that $A = 4/3$. Hence the general solution to our recurrence relation is

$$a_n = C_1 + \left(C_2 + \frac{4n}{3} \right) \cdot 4^n.$$

It remains to insert the initial conditions :

$$n = 0 \Rightarrow a_0 = 2 = C_1 + C_2,$$
$$n = 1 \Rightarrow a_1 = 3 = C_1 + 4\left(C_2 + \frac{4}{3} \right).$$

Solving, we obtain $C_1 = 31/9$, $C_2 = -13/9$. Hence the final answer is

$$a_n = \frac{31}{9} + \left(\frac{4n}{3} - \frac{13}{9} \right) \cdot 4^n.$$

Tenta 22/04/04

The homogeneous equation is

$$a_n - 5a_{n-1} = 0.$$

The characteristic equation for this is

$$x - 5 = 0,$$

which hence has the root $x = 5$. Hence the general solution to the homogeneous equation is

$$a_n^h = C \cdot 5^n.$$

Our guess for a particular solution should have the form

$$a_n^p = A \cdot 4^n + B.$$

Substituting into the recurrence relation, the requirement on A is that

$$A[4^n - 5 \cdot 4^{n-1}] = 4^n,$$

which implies that $A = -4$. The requirement on B is that

$$B - 5B = 2,$$

thus $B = -1/2$. Hence the general solution to our recurrence relation is

$$a_n = C \cdot 5^n - 4^{n+1} - \frac{1}{2}.$$

It remains to insert the initial condition :

$$n = 0 \Rightarrow a_0 = 2 = C - 4 - \frac{1}{2},$$

from which we obtain $C = 13/2$. Hence the final answer is

$$a_n = \frac{13}{2} \cdot 5^n - 4^{n+1} - \frac{1}{2}.$$