Demonstration exercises for week 1

1 (6.4.2 in Biggs) Show that, in any set of 12 integers, there are two whose difference is a multiple of 11.

2 (10.2.1 in Biggs) In Dr. Cynthia Angst's calculus class 32 of the students are boys. Each boy knows 5 of the girls in the class and each girl knows 8 of the boys. How many girls are in the class ?

3 (10.5.2 in Biggs) How many 4-letter words can be made from an alphabet of 10 symbols if there are no restrictions on spelling except that no letter can be used more than once.

4 (10.7.4 in Biggs) In how many ways can we place 8 rooks on a chessboard so that none can attack any other ?

5 (10.7.5 in Biggs) Suppose there are m girls and n boys in a class. What is the number of ways of arranging them in a line so that all the girls are together ?

6 (1.3.7 in Grimaldi) A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if

(a) there are no restrictions
(b) there must be 6 men and 6 women
(c) there must be an even number of women
(d) there must be more women than men
(e) there must be at least 8 men.

7 (11.1.7 in Biggs) Prove the identity

$$\binom{s-1}{0} + \binom{s}{1} + \cdots + \binom{s+n-2}{n-1} + \binom{s+n-1}{n} = \binom{s+n}{n}.$$

8 (11.2.2 in Biggs) Show that when 3 indistinguishable dice are thrown there are 56 possible outcomes. What is the number of possible outcomes if n indistinguishable dice are thrown ?

9 (11.3.2 (iv) in Biggs) Expand $(3+4x)^6$ as a polynomial in x.