1 (15.1.2 in Biggs) The pathways in a formal garden are to be laid out in the form of a *wheel graph* W_n, whose vertex set is $V = \{0, 1, ..., n\}$ and whose edges are

$$\{0,1\},\ \{0,2\},\ \cdots,\ \{0,n\},$$
$$\{1,2\},\ \{2,3\},\ \cdots,\ \{n-1,n\},\ \{n,1\}.$$

Describe a route around the pathways which atarts and ends at vertex 0 and visits every vertex once only.

2 (15.3.1) Is it possible that the following lists are the degrees of all the vertices of a simple graph ? If so, give a pictorial representation of such a graph.

$$(i)\ 2,2,2,3 \qquad (ii)\ 1,2,2,3,4$$
$$(iii)\ 2,2,4,4,4 \qquad (iv)\ 1,2,3,4.$$

3 (15.4.3 in Biggs) Find a Hamilton cycle in the graph formed by the vertices and edges of an ordinary cube.

4 (15.6.2 in Biggs) Determine the chromatic numbers of the following graphs

Picture omitted

5 (15.7.1 in Biggs) Find orderings of the vertices of the cube graph for which the greedy algorithm requires 2,3 and 4 colors respectively.

6 (15.8.5 and 15.8.6 in Biggs) The *k-cube* Q_k is the graph whose vertices are all binary words of length k and whose edges join words which differ in exactly one position. Show that

(i) Q_k is a regular graph of degree k,

(ii) Q_k is bipartite,

(iii) Q_k has a Hamilton cycle for all $k \geq 2$.