CHALMERS | UNIVERSITY OF GOTHENBURG

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Simulation of Random Fields – LP2 2015/16

Project 1

1. The goal of this task is to generate normally distributed random numbers with different methods and compare their performance. You are allowed to use your favorite programming language for the implementation.

a) Implement a (general) linear congruential random number generator. Write two methods that use the parameters of RANDU and of a better random number generator to generate on $[0, 1]$ uniformly distributed random numbers. Generate sufficiently many samples with both options to perform a statistic testing. Report your results. Do you observe any difference?

b) Implement two methods to transform $\mathcal{U}([0, 1])$ distributed random numbers into standard normally distributed ones. Generate sufficiently many samples based on the standard random number generator of your programming environment for $\mathcal{U}([0, 1])$ distributed random numbers and perform statistical tests. Report your results and compare the two methods.

c) Generate standard normally distributed random numbers with all combinations of methods implemented in a) and b) as well as with the standard random number generator for $\mathcal{N}(0, 1)$ distributed random numbers of your programming environment. Compare the samples with respect to performance and quality.

2. The goal of this task is to implement two algorithms to generate efficiently samples of centered isotropic Gaussian random fields on the square $[0, 1)^2$. Let us consider the class of covariance functions

$$C(x, y) = \int_{\mathbb{R}^2} e^{-2\pi i (p, x-y)} \gamma(p) \, \mathrm{d}p, \qquad x, y \in \mathbb{R}^2,$$

with

$$\gamma(p) = (1 + (p_1^{2k} + p_2^{2k})^l)^{-n}$$

for $k, l, n \in \mathbb{N}$.

Please turn!

a) Let $N = (N_1, N_2)$ be the number of grid points in each direction, where N_1 and N_2 are even. Let $M_i := N_i/2$ for $i = 1, 2$. Show that the set

$$\mathcal{L}^N := \{k_1 = 0, \dots, M_1,\ k_2 = 0, \dots, M_2\} \\ \uplus \{k_1 = 1, \dots, M_1 - 1,\ k_2 = M_2 + 1, \dots, 2M_2 - 1\}$$

defines a maximal subset of $\mathcal{K}^N$ such that the random variables $(V_K^N, K \in \mathcal{K}^N)$ are independent.

b) Use your favorite random number generator from Task 1. for standard normally distributed random numbers to implement Algorithm 2.3.1 from the lecture with variable mesh sizes. Generate samples for different choices of k, l, and n as well as different mesh sizes. Compare the results on how they differ.

c) Do the same as in b) but with your choice of Algorithm 2.3.2 or Algorithm 2.3.3.

d) Compare the complexity of both algorithms implemented in b) and c) for a fixed covariance function and varying meshes. What do you observe?

- **Deadline:** Thursday, December 3, 2015, 10:00
- **Webpage:**
  `http://www.math.chalmers.se/~langa/teaching/SimRF15`
- **Requirement:** To pass the course, both projects as well as the final exam have to be passed.
- **Formalities:** You are allowed to work in groups up to two. You are asked to email the code and your solutions to annika.lang ’at’ chalmers.se. In case you have handwritten solutions for the theoretical part, you are also welcome to hand this in at the beginning of the lecture on December 3.