

Numerical Linear Algebra

Homeworks

Notes

- To pass this course you should do **2 compulsory home assignments** before the final exam. Choose any 2 of 4 assignments.
- Homeworks You should do personally (not in groups).
- Sent pdf file with your assignment to my e-mail *larisa@chalmers.se* before deadline (see the course page for deadlines for every home assignment). Handwritten home assignments can be left in the red box located beside my office.

Homework 1

- Let A be an orthogonal matrix. Show that $\det(A) = \pm 1$. Show that if B also is orthogonal and $\det(A) = -\det(B)$ then $A + B$ is singular. **(0.5 b.p.)**
- A matrix is strictly upper triangular if it is upper triangular with zero diagonal elements. Show that if A is strictly upper triangular and has size n by n , then $A^n = 0$. **(0.5 b.p.)**

- 1. (0.5 b.p.)

Let matrices $A, B \in \mathbb{R}^{n \times n}$. Show that $(AB)^T = B^T A^T$ as well as $(AB)^{-1} = B^{-1} A^{-1}$ (here, A and B are nonsingular).

- 2. (0.5 b.p.) Let $A \in \mathbb{R}^{n \times n}$ is s.p.d. matrix. Using Cholesky factorization of A show that $\|x\|_A = (x^T A x)^{1/2}$ defines vector norm.
- 3. (0.5 b.p.) Let $D = \text{diag}(d_1, \dots, d_n) \in \mathbb{R}^{n \times n}$ is the diagonal matrix such that $d_i \neq 0$. Compute condition numbers $\kappa_1(D)$, $\kappa_2(D)$, $\kappa_\infty(D)$.

- 4. **(0.5 b.p.)** Use the Cholesky Algorithm to decide for which α the following matrix is positive definite:

$$A = \begin{bmatrix} \alpha & 2 \\ 2 & 4 \end{bmatrix}$$

Cholesky Algorithm

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for j = 1 to n
  ljj = (ajj -  $\sum_{k=1}^{j-1} l_{jk}^2$ )1/2
  for i = j + 1 to n
    lij = (aij -  $\sum_{k=1}^{j-1} l_{ik} l_{jk}$ ) / ljj
  end for
end for
```

- **(0.5 b.p.)** Transform the matrix A to the tridiagonal form using Householder reflection. Describe all steps of this transformation.

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

- **(0.5 b.p.)** Transform the matrix A to the tridiagonal form using Given's rotation. Describe step-by-step this procedure.

$$\mathbf{A} = \begin{bmatrix} 5 & 4 & 3 \\ 4 & 6 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

Homework 4 (1 b.p.)

Let A be m by n with SVD $A = U\Sigma V^T$. Compute following expressions in terms of U , Σ , and V :

- 1 $(A^T A)^{-1}$
- 2 $(A^T A)^{-1} A^T$
- 3 $A(A^T A)^{-1}$
- 4 $A(A^T A)^{-1} A^T$