Outline to a Decision Support Tool for Traffic Management

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Abstract

We describe the structure and functionality of a decision support tool for strategic traffic management which is currently under development. The tool is intended to be used by traffic planners for determining the actions that need to be taken with respect to the network properties in order to achieve the management goals. The core of the tool is a mathematical traffic equilibrium model that assumes that the travelers choose their routes in accordance with the Wardrop principle. The manager's goals, which typically involve traffic flows and travel times in the network are described by mathematically formulated constraints. The manager also has to specify the actions in the network that are admissible to achieve the goals; examples of such actions are changes in link capacities and monetary tolls.

The proposed approach is a systematic methodology for finding appropriate changes in the traffic network. This is in contrast to the use of a trial-and-error strategy, in which a series of tentative actions in the network are evaluated through the simulation and analysis of each scenario using a traffic equilibrium package. The tool includes a graphical user interface from which it is possible to specify the management goals and the feasible travel time adjustments in the network model. The emphasis of the paper is on the mathematical base for the tool.

1 Introduction

The process of strategic traffic management aims at the improvement of the functionality of the traffic network through modifications of its properties. The functionality under consideration can, for example, involve the traffic flow in a specified zone of the network or the travel time of a sequence of links. The traffic management decisions are often preceded by the formulation (explicitly or implicitly) of targets or *goals,* for the utilization of the traffic network. The manager's goals may be influenced by environmental, safety, practical, political, economical and other considerations. Examples of such goals, all related to the level of traffic flow on the links, are: maximal allowed traffic flow on a link, maximal travel times between certain locations using public transportation, maximal exhaust fume emissions in a certain area, and a sufficient proportion of the trips between an origin and a destination by public transportation.

To reach the management goals, *actions* are taken in the network that result in an altered travel behavior of the travelers. Any action that will alter the travel times (or costs) for the travelers can be used for this purpose. To alter the travel times on links in the network, a number of implementable actions are chosen. Examples of actions that can be used are: adjustments in the setting of traffic signal green times for public and private transportation, altered ticket prices for public transportation, adjustments in speed limits, and modifications in the street network such as speed-bumps. Another possible action is the introduction of road pricing. It is, of course, likely that complementary actions in combination will be the most effective method for reaching the traffic management goals.

Each action causes implemented causes an adjustment of the perceived travel time. Depending on which actions are possible and appropriate to implement on certain links or a certain part of the network, there are restrictions on the levels of these *travel time adjustments* that are possible to achieve. The traffic management process is today often based on repeated scenario analyses. The travel time adjustments that result in the desired traffic flow are then found by trial-and-error adjustments of the travel time functions. Each travel time adjustment is evaluated using a traffic equilibrium package (e.g., Emme/2) and if the adjustment does not produce the desired traffic flow, then the procedure is repeated (see Figure 1).

We propose a systematic methodology for the derivation of travel time adjustments based on mathematical models, and describe the structure and functionality of a decision support tool for traffic management which is currently under development. The tool is based on a mathematical network equilibrium model for demand, route and mode choice and the tool is targeted to the management of an urban traffic network. The core of the tool is a traffic equilibrium model in which, in the most simplistic case, it is assumed that the travelers choose their routes in accordance with the Wardrop principle for fixed demand, static traffic equilibrium assignment. This implies that the goals and actions considered are assumed to be time independent. The proposed mathematical formulation of the above described decision situation requires that the goals can be formulated as restrictions, or *side constraints*, to an equilibrium assignment model. We believe, however, that this requirement is actually advantageous, since the formulation of side constraints to the equilibrium model supports the process of identifying and formulating proper goals.

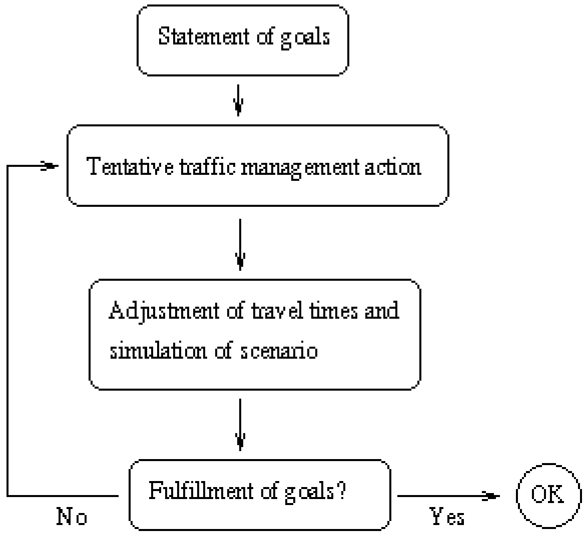

Figure 1: Trial-and-error traffic management procedure

The user of the tool (the traffic manager) also has to identify a set of implementable actions that can be used to enforce the fulfillment of his/her goals. From the set of actions, restrictions on the maximal level of travel time adjustment are constructed. The tool first tries to establish whether or not the formulated goals and the restricted set of travel time adjustments are consistent, that is, if the goals can be attained by the means available. In the case of inconsistency the manager is asked to revise the goals, by the use of a graphical user interface. When the goals and the feasible adjustments are verified to be consistent, the traffic management tool identifies and provides the traffic manager with the necessary travel time adjustments, in order to fulfill the stated goals.

In the following section we first define necessary notation for the equilibrium model. The traffic manager's problem is then formulated as a (nonconvex) bilevel optimization problem, and we present the assumptions which make it possible to simplify the bilevel problem into two convex single-level problems. In Section 3 the cases of inconsistency in the formulated goals and travel time adjustment restrictions are discussed in more detail. In Section 4 the complete tool and the usage thereof is discussed. For simplicity we have in our presentation restricted ourselves to the case of static, single mode network equilibrium. Extensions of the traffic equilibrium model are discussed in the concluding section.

2 The Traffic Management Model

Let the traffic network be modeled by the nodes *N* and links *A*. Assume that the travel demand in all relations in the network are known and independent of the travel times (inelastic demand). Let d_{pq} denote the travel demand between the origin p and the destination q. Let C be the set of travel relations *(p,q)*. Let R_{pq} denote all routes in relation *(p,q)*. The set of route flows *H* is defined by the system

$$\sum_{r \in R_{pq}} h_{pqr} = d_{pq,} \quad \forall (p,q) \in C,$$
$$h_{pqr} \geq 0, \quad \forall r \in R_{pq}, \forall (p,q) \in C.$$

Under the (reasonable) assumption that the network is strongly connected, the set *H* is non-empty and compact. Let $d_{pqra} = 1$ if route *r* from *p* to *q* includes link *a*, and 0 otherwise. The link flow on link *a* is calculated as

$$f_a = \sum_{(p,q) \in C} \sum_{r \in R_{pq}} d_{pqra} h_{pqr}, \quad \forall a \in A.$$

Let *F* be the set of link flows that might result from the route flows in *H*.

We consider a traffic equilibrium which comply with the Wardrop conditions (e.g., Sheffi, 1985). These equilibrium conditions state that every traveler chooses to travel in the fastest possible way where the travel time on link *a* is given by $t_a(f_a)$ where the function t_a is increasing, positive and continuous. The travel time of a route $r \in R_{pq}$ is calculated as $\sum_{a \in A} d_{pqra} t_a(f_a)$. The equilibrium conditions can be formulated as

$$h_{pqr} > 0 \quad \Rightarrow \quad \sum_{a \in A} d_{pqra} t_a(f_a) = p_{pq}, \quad \forall r \in R_{pq},$$
$$h_{pqr} = 0 \quad \Rightarrow \quad \sum_{a \in A} d_{pqra} t_a(f_a) \geq p_{pq}, \quad \forall r \in R_{pq},$$

where p_{pq} can be interpreted as the minimum route cost in relation *(p,q)*. The equilibrium link flow solution can be found by solving the convex optimization problem,

$$\min_{f \in F} \; T(f) = \sum_{a \in A} \int_0^{f_a} t_a(s) ds. \qquad \text{(TAP)}$$

As discussed in the introduction, the manager's goals are often natural to formulate as restrictions on the flow or on the travel times of sets of links and to incorporate the manager's goals in the equilibrium model we require that the goals can be mathematically formulated as constraints in the model. The goals may be determined by environmental, practical and economical constraints, and possibly by other considerations. Assume that the goals are described by the set

$$G = \{ f \mid g_k(f) \leq 0, \quad k = 1, \ldots, K \}$$

where each function g_k is a convex function that describes or quantifies a specific goal. Examples of restrictions that can be formulated as restrictions in the equilibrium assignment model are: restrictions on the traffic flows (e.g., maximal traffic flows on a link), maximal travel times between certain locations using public transportation, maximal exhaust fume emissions in a certain area, and a sufficient proportion of the trips between an origin and a destination by public transportation. We give three examples of manager goals modeled by constraints of the above type.

Flow restriction examples. (a) Suppose that our goal is to restrict the total flow on the road defined by link 2 and 5 shown in Figure 2. The restriction that the maximal allowed total flow is b_1 units is imposed by formulating the constraint $g_1(f) = f_2 + f_5 - b_1 \leq 0$. ■

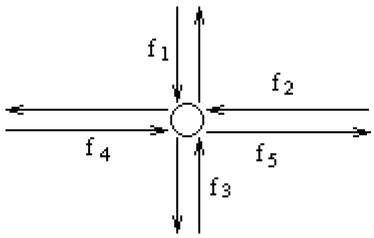

Figure 2: Simple intersection.

(b) Suppose that we would like to restrict the flow through the intersection shown in Figure 2. The inflow is calculated as the sum over the links 1 to 4 in the figure. To restrict this flow to b_2 units, we formulate the constraint $g_2(f) = f_1 + f_2 + f_3 + f_4 - b_2 \leq 0$. ■

Travel time restriction example. This type of constraints can model the total travel time on a route or on a route segment in the network. A restriction that the total travel time on links f_4 and f_5 may not be more that b_3 time units can be formulated as $g_3(f) = t_4(f_4) + t_5(f_5) - b_3 \leq 0$. This type of constraint can, for example, be of interest if the links 4 and 5 represent a bus lane. ■

There are often natural restrictions associated with the chosen set of actions that are necessary to take into consideration. Each of the possible actions are associated with a travel time adjustment which represents the change in travel time perceived by the travelers on the link where the action is implemented. Examples of such travel time adjustment restrictions can be: limitations on how much traffic signal settings on one link can alter the travel time on that link, that a specific link not can (for some reason) be modified in any way and the number of link tolls, their possible placements and toll levels.

Let the vector $r \in \Re^{|A|}$, with elements r_a, represent the travel time adjustment caused by the action taken by the manager. From restrictions on the set of actions that are possible to implement we formulate constraints on the travel time adjustments. We assume that the set of feasible travel time adjustments are described by the set

$$P = \{r \,|\, w_l(r) \le 0, \quad l = 1,\ldots,L\},$$

where each function w_l is assumed to be a convex function.

Travel time adjustment restriction example. In the intersection model (see Figure 2), assume that it is impossible to implement any action that alters the travel time on links 3 and 4, and that the travel time adjustment of an implementable action on link 1 is at least a_1 and at most a_2 time units (which, for example, can be the minimal and maximal acceptable travel time adjustment that can be implemented by a traffic signal on the link). This can be formulated by the constraints $a_1 \le r_1 \le a_2$ and $r_3 = r_4 = 0$. ■

With the possibility to adjust the travel times on the links we can state an equilibrium model with the generalized link travel times given by $t_a(f_a) + r_a$. The Wardrop equilibrium conditions for these travel times can be stated as

$$h_{pqr} > 0 \quad \Rightarrow \quad \sum_{a \in A} d_{pqra}(t_a(f_a) + r_a) = p_{pq}, \quad \forall r \in R_{pq}, \tag{1a}$$

$$h_{pqr} = 0 \quad \Rightarrow \quad \sum_{a \in A} d_{pqra}(t_a(f_a) + r_a) \ge p_{pq}, \quad \forall r \in R_{pq}, \tag{1b}$$

The equilibrium link flow solution that satisfies (1) can be found as a solution to an optimization problem of the same form as (TAP), but with link travel times given by $t_a(f_a) + r_a$. We denote this optimization problem (TAP_r).

The travelers choose their routes according to a criteria which might be in conflict with the management goals. When modeling such a situation using mathematical models, the travelers are often modeled as followers, that is, they re-evaluate their route choices after a change in the traffic network. Mathematically, the interaction between the manager and the travelers may be described as a Stackelberg game.

We formulate the traffic manager’s problem of finding an equilibrium link flow solution, which is feasible with respect to the formulated goal constraints and the travel time adjustment constraints. The managers problem can be formulated as the bilevel problem (e.g., Migdalas, 1995)

$$\min \; j(f, r) \tag{BP}$$

subject to

$$f \in G,$$
$$r \in P,$$
$$f \text{ solves } (TAP_r).$$

The objective function $j(f, r)$ is formulated to specify a secondary management goal (in contrast to the side constraints, which describe primary goals). One example of secondary goal is to minimize the flow-weighted travel time adjustment that is obtained by the objective function $j(f, r) = \sum_{a \in A} f_a r_a$. (In the case where each r_a is a monetary link toll, this function describes the total toll revenue.) Another example is to minimize the system cost that can be formulated with, $j(f, r) = \sum_{a \in A} (t_a(f_a) + r_a) f_a$. The bilevel problem (BP) is in general nonconvex and therefore hard to solve. Several heuristics have been proposed for its solution (e.g., Ferrari, 1999, Cree et al., 1998, and Huang and Bell, 1998). Most of them are computationally very demanding. Earlier attempts to make simplifications by introducing further assumptions of the problem so that the bilevel problem can be reformulated into a single-level problem include Chen et al. (1998).

We next present a scheme for finding an approximate solution to the traffic manager's problem. This solution scheme is a two-stage procedure, and each of the stages amounts to the solution of a convex optimization problem. In the first stage, we find a link flow solution by solving an equilibrium problem where the restrictions formulated from the goals are introduced as side constraints. In stage two, an optimal set of travel time adjustments are sought under the restriction that the equilibrium link flows from stage one is reproduced. The solution to the mathematical model solved in stage two is the vector of travel time adjustments that are necessary to make in order to fulfill the specified goals.

To find a vector r of travel time adjustments such that the solution to (TAP_r) belongs to the set *G,* we can solve the side constrained traffic equilibrium problem

$$\min \; T(f) = \sum_{a \in A} \int_0^{f_a} t_a(s)\, ds \tag{2a}$$

subject to

$$f \in F, \tag{2b}$$

$$g_k(f) \leq 0, \quad k = 1, \ldots, K. \tag{2c}$$

A solution scheme which, after small modifications, can be applied to this problem is found in Larsson et al. (1997). The constraints (2b) and (2c) are not necessarily consistent. How the existence of a feasible solution to this side constrained assignment problem can be ensured is discussed in Section 3. Denote the link flow solution to the side constrained equilibrium problem by f^* and the dual variables to the constraints (2c) by the vector b^*, with elements b_k^*. From the Karush-Kuhn-Tucker conditions for the problem (2) it can be verified (Larsson and Patriksson, 1999, Theorem 2.1) that its solution is a solution to (TAP_r) with r chosen as

$$r_a = \sum_{k=1}^{K} b_k^* \frac{\partial g_k(f^*)}{\partial f_a}, \quad a \in A. \tag{3}$$

From (3) we thus obtain travel time adjustments, r_a, that result in the fulfillment of the formulated goals.

However, these travel time adjustments are not necessarily feasible and implementable, that is, they do not necessarily satisfy the travel time adjustment restrictions (i.e., $r \notin P$ might hold). The dual solution b_k and the associated travel time adjustments are in general not unique (Larsson and Patriksson, 1998, Example 7.5.3). This implies that if the travel time adjustments are not possible to implement we may still be able to find an alternative set of adjustments, which reproduce the same traffic flows and which is feasible. This is stage two of the solution scheme.

We define the set of travel time adjustments that reproduce link flows f^* by a reformulation of the Wardrop conditions (1), with $f = f^*$, into the system

$$\sum_{a\in A} d_{pqra}(t_a(f_a^*)+r_a)-p_{pq} \geq 0, \quad \forall r \in R_{pq},$$

$$\sum_{(pq)\in C}\sum_{r\in R_{pq}} h_{pqr}\left(\sum_{a\in A} d_{pqra}(t_a(f_a^*)+r_a)-p_{pq}\right)=0,$$

where h is any vector of route flows that is consistent with the travel demands and the link flows f^*. This system can equivalently be stated as

$$\sum_{a\in A} d_{pqra}(t_a(f_a^*)+r_a)-p_{pq} \geq 0, \quad \forall r \in R_{pq},$$

$$\sum_{a\in A} d_{pqra}(t_a(f_a^*)+r_a)f_a^* - \sum_{(p,q)\in C} p_{pq}d_{pq} = 0.$$

The optimization problem for finding the feasible travel time adjustments that optimize our secondary management goals, defined by the function $j(f^*, r)$, is then given by

$$\min \; j(f^*, r) \tag{4a}$$

subject to

$$\sum_{a\in A} d_{pqra}(t_a(f_a^*)+r_a)-p_{pq} \geq 0, \quad \forall r \in R_{pq}, \tag{4b}$$

$$\sum_{a\in A} d_{pqra}(t_a(f_a^*)+r_a)f_a^* - \sum_{(p,q)\in C} p_{pq}d_{pq} = 0, \tag{4c}$$

$$w_l(r) \leq 0, \quad l = 1,\ldots,L. \tag{4d}$$

With the aim of reaching our formulated goals with the smallest possible changes in the network, we formulate an objective function which minimizes the total flow-weighted travel time adjustments:

$$j(f^*, r) = \sum_{a\in A} f_a^* \,|\, r_a \,|.$$

(The absolute sign is used because r_a can take on negative values.) The problem (4) is a convex optimization problem. An inherent difficulty of this problem is, however, that the number of constraints in (4b) is, in general, very large. For the solution of this problem we apply a dual solution scheme in combination with constraint generation of the constraints (4b). How the existence of a feasible solution to the travel time adjustment problem is ensured is discussed in the following section.

3 Consistency Considerations

The traffic management goals, which are formulated as constraints and added to the equilibrium assignment model may be overly ambitious, making them impossible to satisfy together with the travel demands. Consequently, we obtain an *infeasible* side constrained equilibrium assignment problem (2). To proceed with the solution scheme we then need information about how the goals need to be weakened in order to make them more conform to the travel demand. (Goals that are so strong that they do not allow the travel demand to be fulfilled are clearly too optimistic or badly formulated, and therefore need to be revised.) In this situation, the tool provides the manager with information about which of the formulated goals that were impossible to fulfill, and for these constraints the level of violation is presented.

Specifically, we solve a (convex) optimization problem for finding the point in F that minimizes the maximal level of violation of the side constraints:

$$\min_{f \in F} \max_{k=1,\ldots,K} g_k(f).$$

Solving this problem does not only to give us a feasible point, if there is one, but the optimal objective value gives us a measure of how difficult the side constraints are to satisfy. The optimal objective value is non-positive for the case of consistency, and positive otherwise, that is if no feasible solution in F conform to the side constraint can be found.

The question of consistency arises also in stage two of the procedure. When the restrictions on the travel time adjustments, derived from the set of feasible actions in the network, is introduced in the travel time adjustment problem (4), this problem may become infeasible. In such a situation the goals are unattainable by the actions that can be taken and the formulated goals or the collection of possible actions need to be revised. The goals that may need to be revised can be monitored based on the (route flow) multipliers for the constraints (4b) within the Lagrangean dual scheme for the problem (4), and the tool provides the same type of information as in the first stage.

4 The Decision Support Tool

In this section we discuss the practical layout of the decision support tool which is based on the proposed approximate solution scheme for the traffic management problem (BP*)*.

The traffic management tool is used in combination with a graphical user interface (GUI), from which it is possible to specify the management goals and the feasible travel time adjustments in the network model. The goals and the network model are first run through the consistency check and the interface reports on the consistency between the travel demands and the management goals (see Figure 3). In the case of inconsistency, the manager is informed of which constraints that result in the inconsistency and to which level the goals are violated.

When consistency has been ascertained the goals are used in the side constrained network assignment, which produces a set of equilibrium link flows which satisfy to the goals. The existence of a feasible travel time adjustment that reproduces these link flows is checked in a consistency procedure. In the case of inconsistency between the equilibrium link flows and the restricted set of travel time adjustments, the goals need to be revised.

Finally, the travel time adjustment procedure results in travel time adjustments that are feasible and that make the goals fulfilled. From these travel time adjustments the manager can then identify a set of actions that can be implemented in the network in order to reach the management goals.

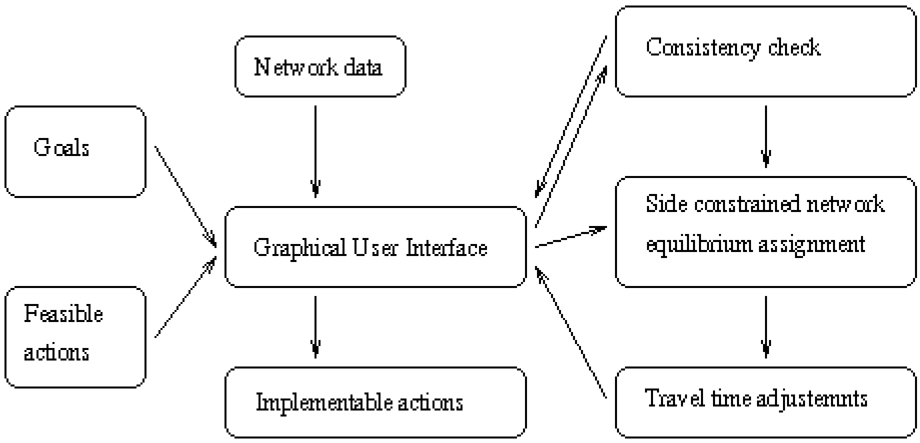

Figure 3: Principal interaction between goals, feasible actions, GUI and the optimization problems.

5 Conclusions

In this paper we have outlined a decision support tool for traffic management which is under development. The approach presented is a systematic methodology for finding changes in the traffic network that are adequate for reaching the management goals. Through the explicit formulation of the goals, the travel time adjustments (associated with implementable actions in the network) necessary to fulfill the goals are provided automatically. This is in contrast to a traditional scenario analyses strategies, in which a series of travel time adjustments is generated by a heuristic trial-and-error technique and evaluated through the use of a traditional traffic equilibrium package.

The presented approach can be extended to more complex equilibrium models without any principle difficulties. Further research will deal with the practical implications of extending the tool to include, for example, elastic demands and several travel modes.

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