Microcalcification Detection in Mammography using Wavelet Transform and Statistical Parameters

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- Introduction
- Wavelet Framework
- One Dimensional Disceret Wavelet Transform
- Two Dimensional Disceret Wavelet Transform
- Microcalcification Detection in Mammography
- Result and Discussion

Introduction

- **Breast cancer** is a cause of cancer death in women. The rates for breast cancer death have been decreasing by earlier detection with specific breast exam called **mammogram** [4].
- A **mammography** is a type of imaging that uses a low-dose x-ray system to examine breasts.
- One of the indicators of breast cancer searched in mammograms are clusters formed by **microcalcifications**.





Wavelet based methods

- In [1]: Ted C. Wang, Detection of microcalcifications in digital mammograms using wavelets.
 - Decimated algorithms for Daubechies wavelet transform (Db2,Db10).
 - Reconstruct only the wavelet coefficients.
 - High number of false positive results.
- In [2]: K.Prabhu, Wavelet based microcalcification detection on mammographic images.
 - Undecimated algorithms for Daubechies wavelet transform (Haar).
 - Microcal detection by the statistics parameters (skewness, kurtosis).
- In [3]: M. Gurcan, Detection of microcalcifications in mammograms using higer order statistics.
 - Undecimated algorithms.

- Microcal detection by statistical test based on skewness and kurtosis quantities .

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• In the present work:

- The decimated algorithm for DWT with 2 null moments is considered.

- For each row and column of the sets of wavelet coefficients, **skewness** and **kurtosis** values are computed.

- The vectors containing these values are then thresholded.

- The crossing of common lines and columns associated to the significant values determine **ROI**.

Wavelet Framework

Definition

Multiresolution Analysis

A multiresolution analysis (MRA) is a family of subspaces $V_j \in L^2(\mathbb{R})$ that satisfies the following properties:

I. Monotonicity

The sequence is increasing, $V_j \subset V_{j+1}$ for all $j \in \mathbb{Z}$.

II. Existence of the Scaling Function

There exists a function $\varphi \in V_0$, such that the set $\{\varphi(.-k) : k \in \mathbb{Z}\}$ is an orthonormal basis for V_0 .

III. Dilation Property

For each j, $f(x) \in V_0$ if and only if $f(2^j x) \in V_j$.

IV. Trivial Intersection Property

$$\bigcap_{j\in\mathbb{Z}}V_j=\{0\}.$$

V. Density

$$\overline{\bigcup_{j\in\mathbb{Z}}V_j}=L^2(\mathbb{R}).$$

 $\bullet ~\forall j,k \in \mathbb{Z},$ the dilation, translation and normalization is given by

$$\varphi_{j,k}(x)=2^{j/2}\varphi(2^jx-k).$$

 For every j ∈ Z, W_j is defined to be the orthogonal complement of V_j in V_{j+1}. It means that

$$V_j \perp W_j$$
, $V_j \oplus W_j = V_{j+1}$.

∃ a function ψ(x) ∈ W₀ such that {ψ(2x − k)}_{k∈Z} is an orthonormal basis for W₀.
According to the MRA properties , the whole collection {ψ_{j,k}; j, k ∈ Z}, is an orthonormal basis for L²(ℝ).

• Scaling and Wavelet equations

The scaling function $\varphi(x) \in V_0$ and the wavelet function $\psi(x) \in V_1$ can be written as:

$$\varphi(x) = \sum_{k \in \mathbb{Z}} h_k \varphi_{1,k}(x) = 2^{1/2} \sum_{k \in \mathbb{Z}} h_k \varphi(2x - k),$$
$$\psi(x) = 2^{1/2} \sum_{k \in \mathbb{Z}} g_k \varphi(2x - k).$$

• A function $f_j \in V_j$ can be splitted into its orthonormal components in V_{j-1}, W_{j-1}

$$Pf(x) = \sum_{l=0}^{N_{j-1}-1} c_{j-1,l} \varphi_{j-1,l}(x) + \sum_{l=0}^{N_{j-1}-1} d_{j-1,l} \psi_{j-1,l}(x),$$

where

$$c_{j-1,l} = \langle f, \varphi_{j-1,l} \rangle, \quad d_{j-1,l} = \langle f, \psi_{j-1,l} \rangle.$$

• Discrete Wavelet Transform

Considering $c_{j,l} = f_j(x_l)$ for $l = 0, \dots, N_j - 1$ and $N_j = 2^{Nmax}$, so

$$c_{j-1,l} = \sum_{k=0}^{D-1} h_{k-2l} c_{j,k}$$
, and $d_{j-1,l} = \sum_{k=0}^{D-1} g_{k-2l} c_{j,k}$,

The normalized Haar scaling filters are:

$$h_0 = 1, \quad h_1 = 1.$$

The normalized **Db2** scaling filters are:

$$h_0 = rac{1+\sqrt{3}}{4}, \quad h_1 = rac{3+\sqrt{3}}{4}, \quad h_2 = rac{3-\sqrt{3}}{4}, \quad h_3 = rac{1-\sqrt{3}}{4}$$

Haar Scaling and Wavelet Functions



Db2 Scaling Function Construction via Cascade Algorithm Iterations



Db2 Wavelet Function Construction via Cascade Algorithm Iterations



• Discrete Inverse Wavelet Transform

The coefficients $c_{j,k}$ can be reconstructed by $c_{j-1,l}$ and $d_{j-1,l}$.

$$c_{j,k} = \sum_{I=\left[\frac{k-D+1}{2}\right]}^{\left[\frac{k}{2}\right]} h_{k-2I}c_{j-1,I}, + \sum_{I=\left[\frac{k-D+1}{2}\right]}^{\left[\frac{k}{2}\right]} g_{k-2I}d_{j-1,I}.$$

• Periodic Extension

Perform an even extension by $f_{n+k} = f_k$ for k > 0, and $f_{-k} = f_{n-k}$ for k < 0 makes the function *periodic*.

• Zero Padding Extension

Add enough zeros to the initial function as $f_k = 0$ for k < 0 and k > n - 1.

Symmetric Extension

The function is extended at the end points by reflection.

• Two Dimensional Scaling and Wavelet Functions

To construct the two dimensional wavelet functions from one dimensional scaling function $\varphi(x)$ and wavelet function $\psi(x)$, we define a *scaling function* $\Phi(x, y)$ by:

$$\Phi(x,y)=\varphi(x)\varphi(y),$$

and three two dimensional wavelet functions as

$$\Psi^{H}(x, y) = \varphi(x)\psi(y),$$

$$\Psi^{V}(x, y) = \psi(x)\varphi(y),$$

$$\Psi^{D}(x, y) = \psi(x)\psi(y).$$

• Dilated, translated, and normalized scaling function is defined by

$$\Phi_{j,k}(x,y) = 2^{j} \Phi(2^{j}x - k_{x}, 2^{j}y - k_{y}),$$

where $j \in \mathbb{Z}$ and $k \in \mathbb{Z}^2$.

The Scaling and the three corresponding Db2 Wavelet Functions



• Two Dimensional Discrete Wavelet Transform

Consider the set of input data represented by the matrix $M = [f_{n,m}]$ where $n, m = 0, \dots, N_k - 1$ and $N_k = 2^{Nmax}$.



• Example 1. Two Dimensional Db2 Wavelet Transform Consider the input matrix $M = [m_{ij}]$ defined by $m_{ij} = i * x_j$ where $x_i = \frac{j}{16}$ for $i, j = 1, 2, \cdots, 16$.

0.37

0.68

1.3

2 2.2

4.1 4.5

2.7

4.8

5.5

6.1

6.8

7.5

8.2

8.9

9.6

10.3

11

. . .

. . .

. . .

0.7

1.5

3

5.2 5.6

6

6.7

7.5

8.2

9

9.7

10.5

11.2

12

3.4 3.7

0.81

1.6

2.4

3.2

4

4.8

6.5

7.3

8.1

8.9

9.7

10.5

11.3

12.1

13

0.87

1.7

2.6

3.5

4.3

5.2

6.1

7

7.8

87

9.6

10.5

11.3

12.2

13.1

14

0.93 1

1.8 2

2.8

3.7

4.6

5.6

6.5

7.5 8

8.4 9

9.3 10

10.3 11

11.2 12

12.113

13.1 14

14 15

15 16

 $M = \begin{bmatrix} 0.06 & 0.12 & 0.18 & 0.25 & 0.31 & 0.37 \\ 0.12 & 0.25 & 0.37 & 0.5 & 0.62 & 0.7 \\ 0.18 & 0.37 & 0.56 & 0.7 & 0.93 & 1.1 \\ 0.25 & 0.5 & 0.7 & 1 & 1.2 & 1.5 \\ 0.3 & 0.6 & 0.9 & 1.2 & 1.5 & 1.8 \\ 0.37 & 0.7 & 1.1 & 1.5 & 1.8 & 2.2 \\ 0.43 & 0.8 & 1.3 & 1.7 & 2.1 & 2.6 \\ 0.5 & 1 & 1.5 & 2 & 2.5 & 3 \\ 0.56 & 1.1 & 1.6 & 2.2 & 2.8 & 3.3 \\ 0.62 & 1.2 & 1.8 & 2.5 & 3.1 & 3.7 \\ 0.68 & 1.3 & 2 & 2.7 & 3.4 & 4.1 \\ 0.7 & 1.5 & 2.2 & 3 & 3.7 & 4.5 \\ 0.8 & 1.6 & 2.4 & 3.2 & 4 & 4.8 \\ 0.87 & 1.7 & 2.6 & 3.5 & 4.3 & 5.2 \\ 0.9 & 1.8 & 2.8 & 3.7 & 4.6 & 5.6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$

0.06

0.12

0.18

3

0.25

0.31

Microcalcification Detection in Mammograph

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Figure: Function $M = [m_{ij}]_{i,j \in \mathbb{N}}$, defined $m_{ij} = i * x_j$ where $x_j = \frac{j}{16}$ for i = 1 : 16, j = 1 : 16.

• Example 1.1

In this example two issues are investigated:

(1) What happen with coefficients near the boundaries.

(2) What happen with the wavelet coefficients in each one of the three blocks away from the boundaries.

 $\widehat{M} = 2DWT(M)$.

	0.6	1.4	2.3	 4.7	5.5	5.9	0	0	0	 0	0	-1.6 7
	1.4	3.3	5.1	 10.5	12.3	13.2	0	0	0	 0	0	-3.6
	2.3	5.1	7.9	 16.3	19.2	20.5	0	0	0	 0	0	-5.6
	3.1	6.9	10.7	 22.2	26	27.7	0	0	0	 0	0	-7.6
	3.9	8.7	13.5	 28	32.8	35	0	0	0	 0	0	-9.6
	4.7	10.5	16.3	 33.8	39.6	42	0	0	0	 0	0	-1.6
	5.5	12.3	19.2	 39.6	46.4	49.6	0	0	0	 0	0	-133.6
\widehat{M} –	5.9	13.2	20.5	 42.3	49.6	53	0	0	0	 0	0	-14.5
101 —	0	0	0	 0	0	0	0	0	0	 0	0	0
	0	0	0	 0	0	0	0	0	0	 0	0	0
	0	0	0	 0	0	0	0	0	0	 0	0	0
	0	0	0	 0	0	0	0	0	0	 0	0	0
	0	0	0	 0	0	0	0	0	0	 0	0	0
	0	0	0	 0	0	0	0	0	0	 0	0	0
	0	0	0	 0	0	0	0	0	0	 0	0	0
	-1.6	-3.6	-5.6	 -11.6	-13.6	-14.5	0	0	0	 0	0	4

Microcalcification Detection in Mammograph

• Example 1.2

Consider $N = [n_{i,j}]$,

$$n_{i,j} = \begin{cases} m_{i,j} & i, j \neq 8 \\ m_{i,j} + 100 & i, j = 8 \end{cases}$$

By Decompose N ($\hat{N} = 2DWT(N)$), what happend with the wavelet coefficients on the three blocks? Do they change in a specific position?

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• Example 1.3

D

we change just the one value, $d_{3,5}^D = 100$, in decomposed matrix \hat{N} and denote the altered matrix by \hat{D} , then we apply the bi-dimensional inverse wavelet transform ($\mathbf{D} = 2DIWT(\hat{D})$) to indicate the effect of this change in reconstruction process.

	0.06	0.12	0.18	 0.5	0.56	0.62	0.68	0.7	0.81	0.87	0.93	1
	0.12	0.25	0.37	 1	1.1	1.2	1.3	1.5	1.6	1.7	1.8	2
	0.18	0.37	0.5	 1.5	1.6	1.8	2	2.2	2.4	2.6	2.8	3
	0.25	0.5	0.7	 2	2.2	2.5	2.7	3	3.2	3.5	3.7	4
	0.3	0.6	0.9	 2.5	3.6	4.5	-1.9	6.8	4	4.3	4.6	5
	0.37	0.7	1.1	 3	4.8	6.2	-5.2	9.9	4.8	5.2	5.6	6
	0.43	0.8	1.3	 3.5	-1.4	-5	39.8	-14.9	5.6	6.1	6.5	7
_	0.5	1	1.5	 4	7.6	10.4	-14.3	17.6	6.5	7	7.5	8
_	0.56	1.1	1.6	 4.5	5	5.6	6.1	6.7	7.3	7.8	8.4	9
	0.62	1.2	1.8	 5	5.6	6.2	6.8	7.5	8.1	8.7	9.3	10
	0.68	1.3	2	 5.5	6.1	6.8	7.5	8.2	8.9	9.6	10.3	11
	0.7	1.5	2.2	 6	6.7	7.5	8.2	9	9.7	10.5	11.2	12
	0.8	1.6	2.4	 6.5	7.3	8.1	8.9	9.7	10.5	11.3	12.1	13
	0.87	1.7	2.6	 7	7.8	8.7	9.6	10.5	11.3	12.2	13.1	14
	0.9	1.8	2.8	 8.4	9.3	10.3	11.2	12.1	13.1	14	15	
	1	2	3	 8	9	10	11	12	13	14	15	16

Thresholding

• **Soft Thresholding** The soft thresholding method on the wavelet coefficients $d_{i,k}^i$ can be performed as:

$$s_{j,k}^{i} = \begin{cases} d_{j,k}^{i} - \lambda^{i} & \text{if } d_{j,k}^{i} > \lambda^{i} \\ d_{j,k}^{i} + \lambda^{i} & \text{if } d_{j,k}^{i} < -\lambda^{i} \\ 0 & otherwise, \end{cases}$$

• Hard thresholding Hard thresholding is another filtering method that is applied on the wavelet coefficients in the following way:

$$m{s}^i_{j,k} = egin{cases} d^i_{j,k} & ext{if} \quad |d^i_{j,k}| \geq \lambda^i \ 0 & ext{if} \quad |d^i_{j,k}| < \lambda^i, \end{cases}$$

where $s_{j,k}^i$ are the threshold wavelet coefficients, and $\lambda^i = \mu^i + \alpha \sigma^i$ is the threshold value.

• Example 2: According to [1]





Figure: Edge detection using the soft, modified soft and hard thresholding method.

Statistical Parameters

• **Skewness** For a sample of *n* values, skewness (S) is the third order correlation parameter defined as:

$$S = \frac{\frac{1}{n} \sum_{l=1}^{n} (x_l - \overline{x})^3}{(\frac{1}{n} \sum_{l=1}^{n} (x_l - \overline{x})^2)^{3/2}},$$

where \overline{x} is the sample mean.



Kurtosis

For a sample of n values, kurtosis (K) is the fourth order correlation parameters defined as:

$$K = \frac{\frac{1}{n} \sum_{l=1}^{n} (x_{l} - \overline{x})^{4}}{(\frac{1}{n} \sum_{l=1}^{n} (x_{l} - \overline{x})^{2})^{2}} - 3,$$



• W Coeffs Histogram of the Mammography with Microcals



• W Coeffs Histogram of the Mammography without microcals



- Numerical Experiments of Skewness
- Example 3, staistical parameters [2,3]
 - Impute image (I).
 - WT(I)=(C,V,H,D).
 - $S^{r}(V), S^{r}(H), S^{r}(D).$
 - $S^{c}(V), S^{c}(H), S^{c}(D).$
 - Threshold of $S^{c}(.), S^{r}(.)$.
 - The significant rows and columns are obtained.
 - Intersections of them detect regions.

Subbands Skewness Case with Calcifications



Subbands Skewness Case without Calcifications



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• Numerical Experiments of Kurtosis

• Example 4, staistical parameters [2,3]

- Impute image (I).
- WT(I) = (C, V, H, D).
- $K^r(V), K^r(H), K^r(D)$.
- $K^{c}(V), K^{c}(H), K^{c}(D).$
- Threshold of $K^{c}(.), K^{r}(.)$.
- The significant rows and columns are obtained.
- Intersections of them detect regions.

vertical row kurtosis horizontal row kurtosis diagonal row kurtosis vali vertical row kurtosis values ule, row kurtosis horizontal number of samples 40 60 80 1 number of samples number of samples diagonal column kurtosis horizontal column kurtosis vertical column kurtosis values 30 107 to sis vertical column kurtosis norizontal column column 7 number of samples number of samples number of samples

Subbands Kurtosis Case with Calcifications

Subbands Kurtosis Case without Calcifications



Here the microcalcifications detection method is posed as a hypothesis testing problem in which the null hypothesis, H_0 , corresponds to the case of no microcalcifications against the alternative H_1 , and it follows the rule Γ based on skewness and kurtosis values,

$$\Gamma(x) = egin{cases} 0 & S_i < T_i & ext{or} & K_i < T_i \ 1 & S_i \geq T_i & ext{and} & K_i \geq T_i, \end{cases}$$

where T_i is the threshold values determined slightly below the maxima of the row and column skewness and kurtosis values of each subband.



- Now we apply the aforesaid algorithms without performing any wavelet transforms in order to investigate if a previous filtering stage of an image is necessary to detect microcalcifications.
 - Inpute image (I).
 - $S^{r}(I), S^{c}(I).$
 - Threshold $S^{r}(I), S^{c}(I)$.
 - $K^{r}(I), K^{c}(I).$
 - Threshold $K^{r}(I), K^{c}(I)$.
 - Perform statistical test.
 - The significant rows and columns are obtained.
 - Intersections of them detect regions.

Result and Discussion

 Tow regions of interest selected calculating skewness and kurtosis values of wavelet coefficients.

 A region selected by the analysis of skewness and kurtosis computed directly from the image data.



Result of the statistical test based on skewness and kurtosis on 24 digitized mammographies from [7].

				mmograms			
			Image	Detection		S, K detections	Identified
				skewness	kurtosis		
			7	1	1	same	1
		8	4	4	same	4	
			9	1	1	same	1
lorn	nal mammogr	raphies	10	2	2	same	1
e	Deteo	Detection		2	2	same	2
	skewness	kurtosis	12	1	1	same	1
	0	0	13	2	2	same	2
	0	0	14	2	2	same	1
	0	0	15	4	4	same	1
	0	0	16	4	4	same	2
	0	0	17	4	4	same	2
	0	0	18	2	2	same	2
			19	1	1	same	1
			20	6	6	same	6
			21	3	3	same	1
			22	2	2	same	1
			23	2	2	same	1
			24	4	4	same	1

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References

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