Applied Mathematics/Partial Differential Equations, part A, for K, Kf, Bt

Problems, IV

Triangulation

1. Consider the triangulation of the unit square $\Omega = [0, 1] \times [0, 1]$ into 8 triangles drawn in Figure 1.

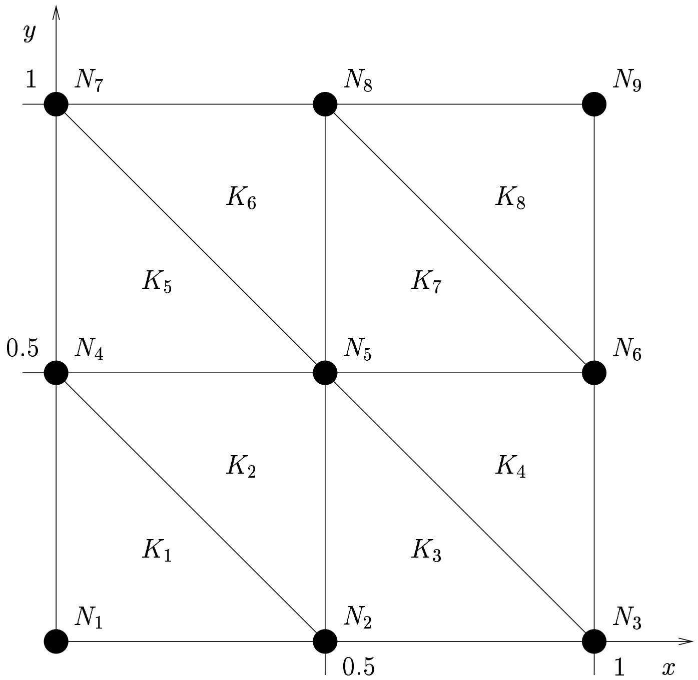

Figure 1: The triangulation in Problem 1.

(a) Compute the length of the largest side h_{K_j}, and the smallest angle τ_{K_j} of the triangles.
(b) Determine the *point matrix* `p` that describes this triangulation in Matlab. *Hint:* Since node 1 is located at the origin, the first column in `p` is `[0; 0]`.
(c) Determine the *triangle matrix* `t` that describes this triangulation in Matlab. *Hint:* Since triangle 1 has corners in node number 1, 2 and 4, the first column in `t` can e.g. be `[1; 2; 4]`. It is not important which node comes first, but they must be listed in a *counter-clockwise* order.
(d) Verify your results by creating `p` and `t` in Matlab:

```
>> p(:, 1) = [0; 0]
>> p(:, 2) = ...
...
>> p(:, 9) = ...
```

```
>> t(:, 1) = [1; 2; 4]
>> t(:, 2) = ...
...
>> t(:, 8) = ...
```

and plot the triangulation by the Matlab-command:

```
>> pdemesh(p, [], t)
```

Continuous Piecewise Linear Functions.

2. Consider the same triangulation as in Problem 1.
(a) The continuous piecewise linear function $\varphi_2(x, y)$ is defined by:

$$\varphi_2(N_2) = 1; \quad \varphi_2(N_j) = 0 \text{ for } j \neq 2.$$

Compute the analytical expression for φ_2. *Hint:* The analytical expressions on K_1, K_2 and K_3 may be determined by solving linear systems of equations as you have seen in the lecture. On the other triangles, $\varphi_2 \equiv 0$. Why?
(b) Plot φ_2 in Matlab by giving the command:

```
>> pdesurf(p, t, [0; 1; 0; 0; 0; 0; 0; 0; 0])
```

or

```
>> pdemesh(p, [], t, [0; 1; 0; 0; 0; 0; 0; 0; 0])
```

Try both! The argument `[0; 1; 0; 0; 0; 0; 0; 0; 0]` is a *column vector* containing the *nodal values* of φ_2. Try also to plot some other "tent functions" φ_j!
(c) Since an arbitrary continuous piecewise linear function v can be written as a linear combination of "tent functions":

$$v(x, y) = v(N_1)\,\varphi_1(x, y) + \ldots + v(N_9)\,\varphi_9(x, y)$$

the "tent functions" $\{\varphi_i\}_{i=1}^{9}$ form a *basis* for the vector space V_h of continuous piecewise linear functions on the triangulation in Figure 1. What is the *dimension* of V_h?
(d) Try plotting some different functions in V_h using the Matlab commands `pdesurf` and `pdemesh`. *Hint:* Cf. how you plotted φ_2.