

# TMA682, Extra övningar i Laplacetransform

Use Laplace transforms to solve the following initial-value problems:

- $y'' - y = 1, \quad y(0) = 0, \quad y'(0) = 1.$   
answer:  $y = e^t - 1.$
- $y'' - 3y' + 2y = 0, \quad y(0) = 3, \quad y'(0) = 4.$   
answer:  $y = e^{2t} + 2e^t.$
- $4y'' + y = -2, \quad y(0) = 0, \quad y'(0) = 1/2.$   
answer:  $y = -2 + 2 \cos(t/2) + \sin(t/2).$
- $y'' + 2y' + y = e^t, \quad y(0) = 0, \quad y'(0) = 0.$   
answer:  $y = \frac{1}{4}e^t - \frac{1}{4}e^{-t} - \frac{1}{2}te^{-t}.$
- $y'' + 2y' + 3y = 3t, \quad y(0) = 0, \quad y'(0) = 1.$   
answer:  $y = \frac{2}{3}e^{-t} \cos \sqrt{2}t + \frac{\sqrt{2}}{3}e^{-t} \sin \sqrt{2}t + t - \frac{2}{3}.$

Find the inverse Laplace transform of the following functions:

- $\frac{1}{s(s+2)^2}$     answer:  $\frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}te^{-2t}.$
- $\frac{1}{s^2+4s+29}$     answer:  $\frac{1}{5}e^{-2t} \sin 5t.$
- $\frac{2s}{(s^2+1)^2}$     answer:  $t \sin t.$
- $\frac{3s^2}{(s^2+1)^2}$     answer:  $\frac{3}{2} \sin t + \frac{3}{2}t \cos t.$
- $\ln \frac{s+3}{s+2}$     answer:  $\frac{1}{t}(e^{-2t} - e^{-3t}).$